Fundamentals of magnetism

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Slides: http://fruchart.eu/slides

My purpose here

- Personal and light overview
- Keep in mind main ideas not detailed concepts and maths
- Not a research talk

Some references – Books

Some references – Repository of ESM

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Non sécurisé | magnetism.eu/esm/repository-topics.html

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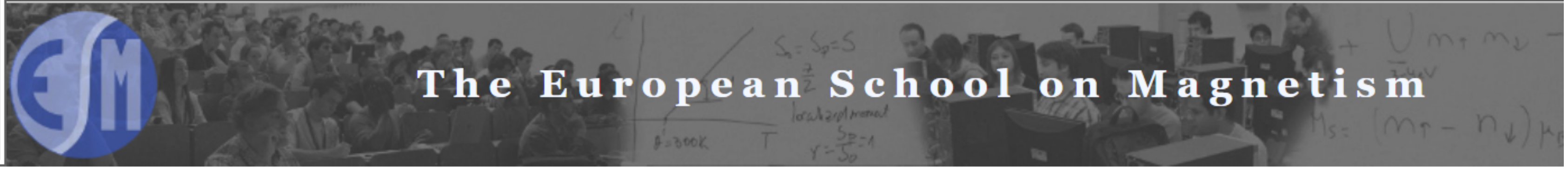


ESM repository | Home | Search | By topics | By authors

The lectures of all ESM schools since 2003 are ordered here in terms of topics. Those pertaining to several topics are listed several times. The topics are:

Magnetic field and moments

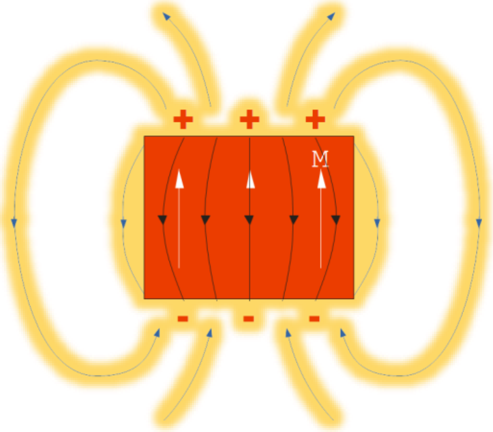
- **[2020]** Origin of magnetism (spin and orbital momentum, atoms and ions, paramagnetism and diamagnetism): **Stephen Blundell**, *Oxford, UK* [Slides | Recording]
- **[2020]** Fields, moments, units: **Olivier Fruchart**, *Grenoble, France* [Slides | Recording]
- **[2019]** Fields, moments, units: **Olivier Fruchart**, *Grenoble, France* [Abstract | Slides]
- **[2019]** Magnetism of atoms, Hund's rules, spin-orbit in atoms: **Virginie Simonet**, *Grenoble, France* [Abstract]
- **[2018]** Units in Magnetism (practical): **Olivier Fruchart**, *Grenoble, France* [Questions | Answers]
- **[2018]** Magnetism of atoms and ions: **Janusz Adamowski**, *Kraków, Poland* [Abstract | Slides]
- **[2018]** Fields, Moments, Units, Magnetostatics: **Richard Evans**, *York, UK* [Abstract | Slides]
- **[2017]** Fields, Units, Magnetostatics: **Laurent Ranno**, *Grenoble, France* [Abstract | Slides]
- **[2017]** Magnetism of atoms and ions: **Wulf Wulfhekel**, *Karlsruhe, Germany* [Abstract | Slides]
- **[2017]** Units in Magnetism (practical): **Olivier Fruchart**, *Grenoble, France* [Questions | Answers]
- **[2015]** Units in Magnetism (practical): **Olivier Fruchart**, *Grenoble, France* [Questions | Answers]

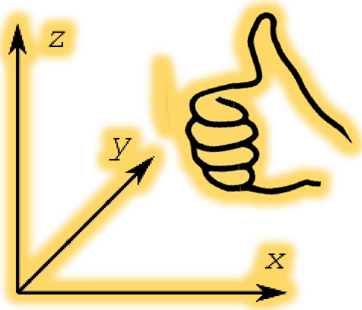
Topics

- Units, fields and moments
- Exchange, magnetic ordering, magnetic anisotropy
- Temperature effects and excitations
- Correlated systems
- Transport
- Magnetization processes
- Simulations
- Materials
- Nanoparticles, microstructures etc
- Nanomagnetism and spintronics
- Techniques
- Applications and interdisciplinary magnetism
- Industry perspectives
- Open sessions

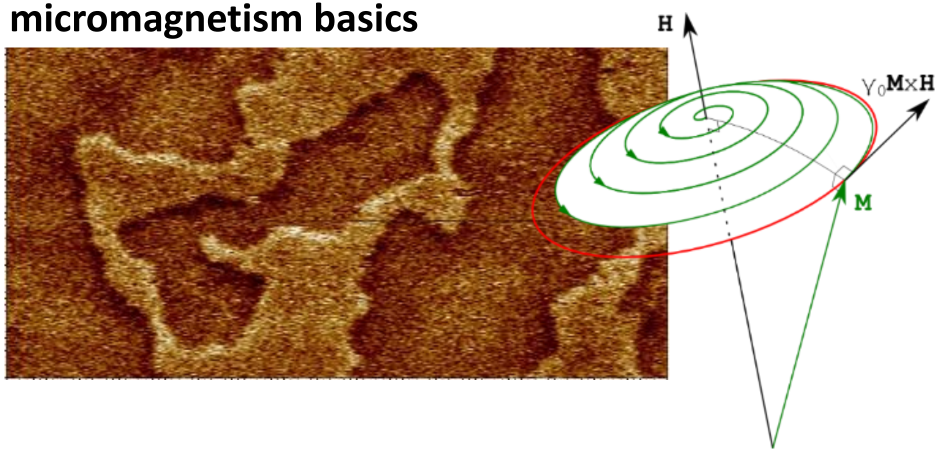
Table of contents (1/3)

- Units and fields

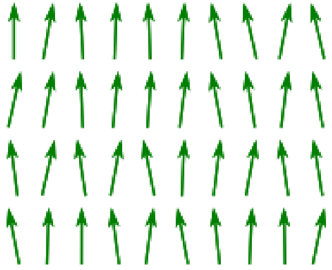




- Magnetization processes and micromagnetism basics

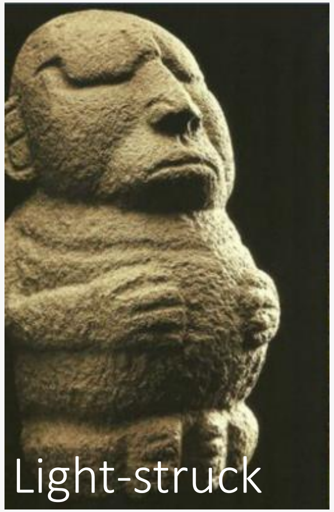


- Magnetic ordering and materials



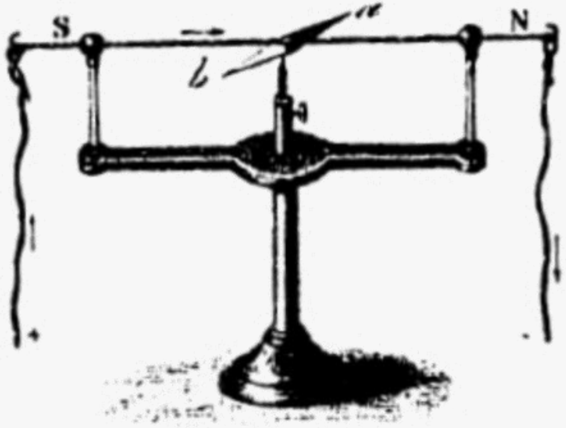
Century-old facts

- Magnetic materials (rocks)



- Magnetic field of the earth

Œrsted experiment in 1820



Hans-Christian Oersted, 1777–1851.

→ Birth of electromagnetism

Maxwell equations (in vacuum)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$ → Gauss theorem

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ → Faraday law of induction

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$ → Ampère theorem

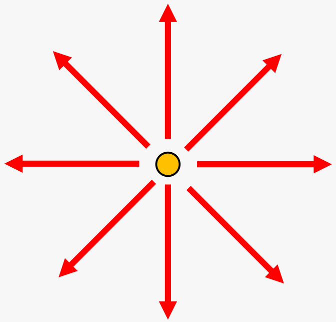
$$\nabla \cdot \mathbf{B} = 0$$ → B is divergence free (no magnetic poles)

The electric charge and the electric field

Macroscopic level: Gauss theroem

- Ostogradski theorem

$$\iiint_{\mathcal{V}} \boldsymbol{\nabla}\cdot\mathbf{E}\,\mathrm{d}\mathcal{V} = \oiint_{\partial\mathcal{V}} \mathbf{E}\cdot\mathbf{n}\,\mathrm{d}\mathcal{S}$$



$$\Rightarrow \quad \frac{Q}{\epsilon_0} = \iiint_{\mathcal{V}} \frac{\rho}{\epsilon_0}\,\mathrm{d}\mathcal{V} = \oiint_{\partial\mathcal{V}} \mathbf{E}\cdot\mathbf{n}\,\mathrm{d}\mathcal{S}$$

Link

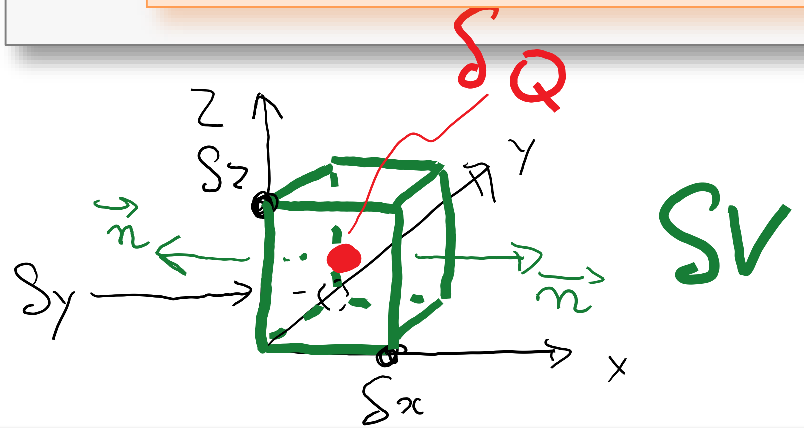
$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{\partial E_x}{\partial x} + \ldots = \frac{E_x(x+\delta x) - E_x(x)}{\delta x} + \cdots$$

Microscopic level: Maxwell equation

$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\rho = \frac{\delta Q}{\delta\mathcal{V}}$$ Volume density of electric charge

- ρ is the scalar source of **E**



The electric current and the magnetic induction field

Macroscopic level: Ampere theorem

- Stokes theorem

$$\iint_{\mathcal{S}} (\boldsymbol{\nabla} \times \mathbf{B}) \cdot \mathbf{n} \, \mathrm{d}\mathcal{S} = \oint_{\partial\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell}$$

$$\Rightarrow \quad I = \mu_0 \iint_{\mathcal{S}} (\mathbf{j} \cdot \mathbf{n}) \, \mathrm{d}\mathcal{S} = \oint_{\partial\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell}$$

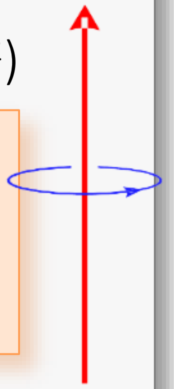
Microscopic level: Maxwell equation

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{j}$$

j: Volume density of current ($\mathrm{A/m^2}$)

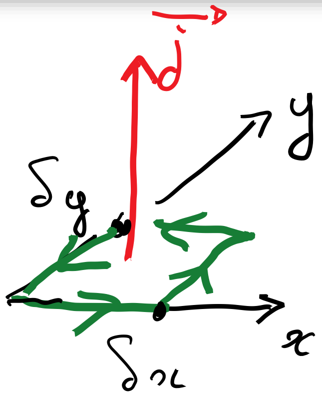
- j is the vectorial source of curl of **B**

Unit for B: tesla (T)



Link

$$\boldsymbol{\nabla} \times \mathbf{B} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{\partial B_y}{\partial x} - \dfrac{\partial B_x}{\partial y} \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{B_y(x+\delta x) - B_y(x)}{\delta x} - \dfrac{B_x(y+\delta y) - B_x(y)}{\partial y} \end{pmatrix}$$



The magnetic point dipole

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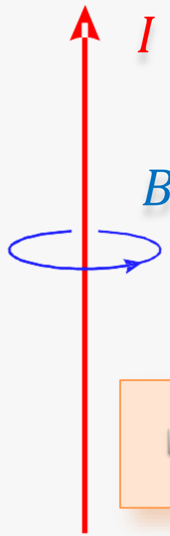
Biot and Savart

$$\delta\mathbf{B} = \frac{\mu_0 I \delta\boldsymbol{\ell} \times \mathbf{u}}{4\pi r^2}$$

- Note: $1/r^2$ decay

Ampere theorem and Œrsted field

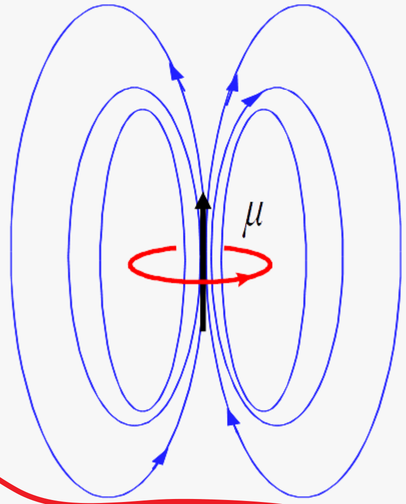


$$B_\theta = \frac{\mu_0 I}{2\pi r}$$

Integrate

- Note: $1/r$ decay

The magnetic point dipole



- Simple loop

$\boldsymbol{\mu} = I\mathcal{S}\,\mathbf{n}$ **Unit**: $\mathrm{A}\cdot\mathrm{m}^2$

- General definition

$$\boldsymbol{\mu} = \frac{1}{2}\iiint_{\mathcal{V}} \mathbf{r}\times\mathbf{j}(\mathbf{r})\,\mathrm{d}\mathcal{V}$$

Derive

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left[\frac{3}{r^2}(\boldsymbol{\mu}\cdot\mathbf{r})\mathbf{r} - \boldsymbol{\mu}\right]$$

- Note: $1/r^3$ decay

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left(2\mu\cos\theta\,\mathbf{u}_r + \mu\sin\theta\,\mathbf{u}_\theta\right)$$

The magnetic point dipole in a magnetic induction field

Energy

$\mathcal{E} = -\boldsymbol{\mu} \cdot \mathbf{B}$ Zeeman energy (J)

Demonstration

- Work to compensate Lenz law during rise of B
- Integrate torque from Laplace force while flipping dipole in B

Torque

$$\boldsymbol{\Gamma} = \oint \mathbf{r} \times I(\mathrm{d}\boldsymbol{\ell} \times \mathbf{B}) = \boldsymbol{\mu} \times \mathbf{B}$$

- Inducing precession of dipole around the field
- It is energy-conservative, as expected from Laplace (Lorentz) force

Force

$\mathbf{F} = \boldsymbol{\mu} \cdot (\overline{\overline{\nabla \mathbf{B}}})$

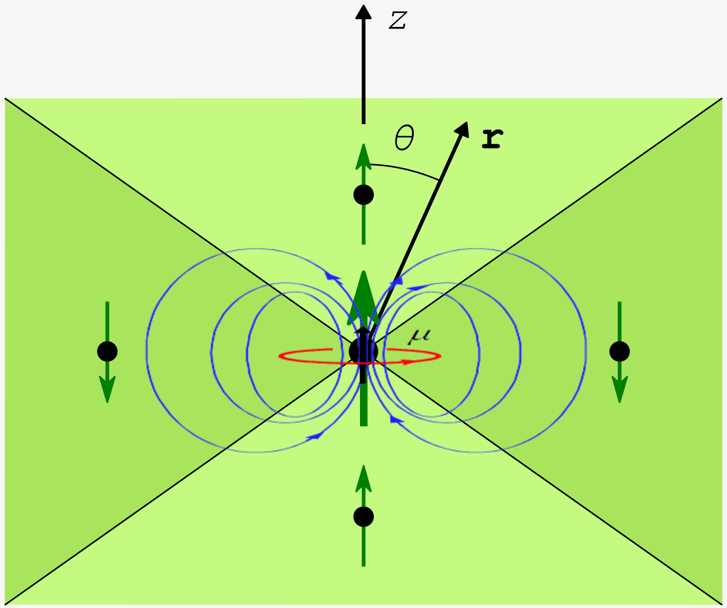
- Valid only for fixed dipole
- No force in uniform magnetic induction field

Two interacting magnetic point dipoles

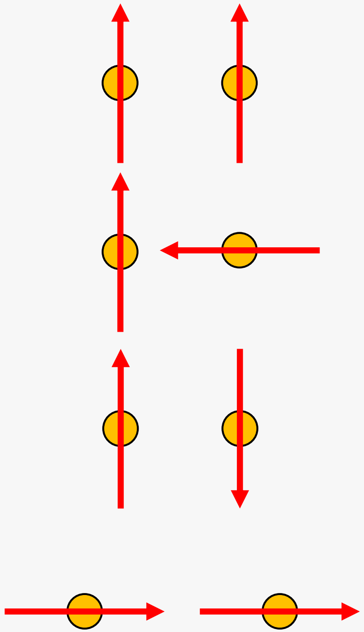
Energy

$$\mathcal{E} = -\frac{\mu_0}{4\pi r^3}\left[\frac{3}{r^2}(\boldsymbol{\mu}_1\cdot\mathbf{r})(\boldsymbol{\mu}_2\cdot\mathbf{r}) - \boldsymbol{\mu}_1\cdot\boldsymbol{\mu}_2\right]$$

- The dipole-dipole interaction is anisotropic



Examples



$$\mathcal{E} = +2\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = +\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = 0$$

$$\mathcal{E} = -\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = -2\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

Magnetization

Definition

- Volume density of magnetic point dipoles

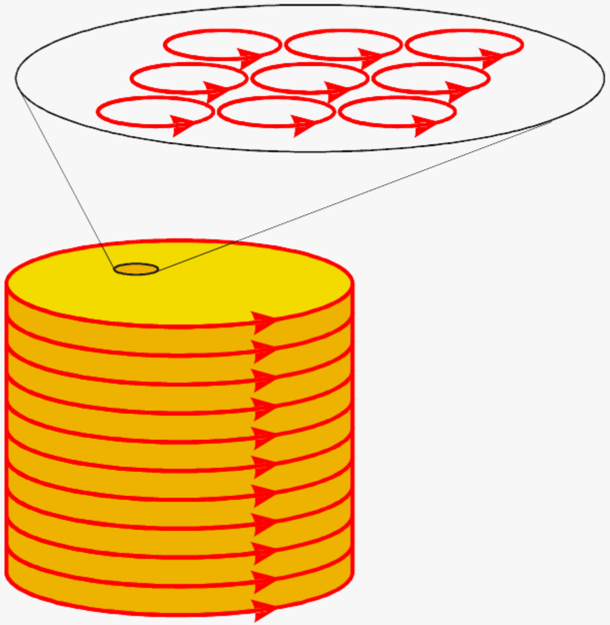
$$\mathbf{M} = \frac{\delta \boldsymbol{\mu}}{\delta \mathcal{V}} \qquad \mathrm{A/m}$$

- Total magnetic moment of a body

$$\boldsymbol{\mathcal{M}} = \int_{\mathcal{V}} \mathbf{M}\,\mathrm{d}\mathcal{V} \qquad \mathrm{A \cdot m^2}$$

- Applies to: ferromagnets, paramagnets, diamagnets etc.
- Must be defined at a length scale much larger than atoms
- Is the basis for the micromagnetic theory

Equivalence with surface currents



- Name: Amperian description of magnetism
- Surface current equals magnetization $\mathrm{A/m}$

Free currents and bound currents

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Back to Maxwell equations

- Disregard fast time dependence: magnetostatics

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \cancel{\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}} \right)$$

- Consider separately real charge current, $\mathbf{j}_\mathrm{c}$ from fictitious currents of magnetic dipoles $\mathbf{j}_\mathrm{m}$

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{j}_\mathrm{c} + \mathbf{j}_\mathrm{m})$$

- One can show: $\nabla \times \mathbf{M} = \mathbf{j}_\mathrm{m}$ $\quad \mathrm{A/m^2}$

  $\mathbf{M} \times \mathbf{n} = \mathbf{j}_\mathrm{m,s}$ $\quad \mathrm{A/m}$

- Outside matter, $\mathbf{B}$ and $\mu_0 \mathbf{H}$ coincide and have exactly the same meaning.

The magnetic field H

- One has: $\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{j}_\mathrm{c}$

- By definition: $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ $\quad \mathrm{A/m}$

$$\nabla \times \mathbf{H} = \mathbf{j}_\mathrm{c}$$

B versus H : definition of the system

- **M**: local (infinitesimal) part in $\delta\mathcal{V}$ of the system defined when considering a magnetic material

- **H**: The remaining of **B** coming from outside $\delta\mathcal{V}$, liable to interact with the system

Derivation of the dipolar field

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The dipolar field H_d

- By definition: the contribution to H not related to free currents (possible to split as Maxwell equations are linear)

$$\nabla \times \mathbf{H}_\mathrm{d} = 0 \Rightarrow \mathbf{H}_\mathrm{d} = -\nabla \phi_\mathrm{d}$$

$$\mathbf{H} = \mathbf{H}_\mathrm{d} + \mathbf{H}_\mathrm{app}$$

External to magnetic body

Analogy with electrostatics

$$\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = -\nabla \phi$$

→ Magnetic scalar potential

Derive the dipolar field

Maxwell equation $\nabla \cdot \mathbf{B} = \mathbf{0} \rightarrow \nabla \cdot \mathbf{H}_\mathrm{d} = -\nabla \cdot \mathbf{M}$

Source for $\mathbf{H}_\mathrm{d}$

$$\Rightarrow \mathbf{H}_\mathrm{d}(\mathbf{r}) = -M_\mathrm{s} \iiint_{\mathcal{V}'} \frac{[\nabla \cdot \mathbf{m}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{V}'$$

To lift the singularity that may arise at boundaries, a volume integration around the boundaries yields:

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \iiint \frac{\rho(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{V}' + \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{S}'$$

$\rho(\mathbf{r}) = -M_\mathrm{s}\, \nabla \cdot \mathbf{m}(\mathbf{r})$ → volume density of magnetic charges

$\sigma(\mathbf{r}) = M_\mathrm{s}\, \mathbf{m}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r})$ → surface density of magnetic charges

Stray field and demagnetizing field

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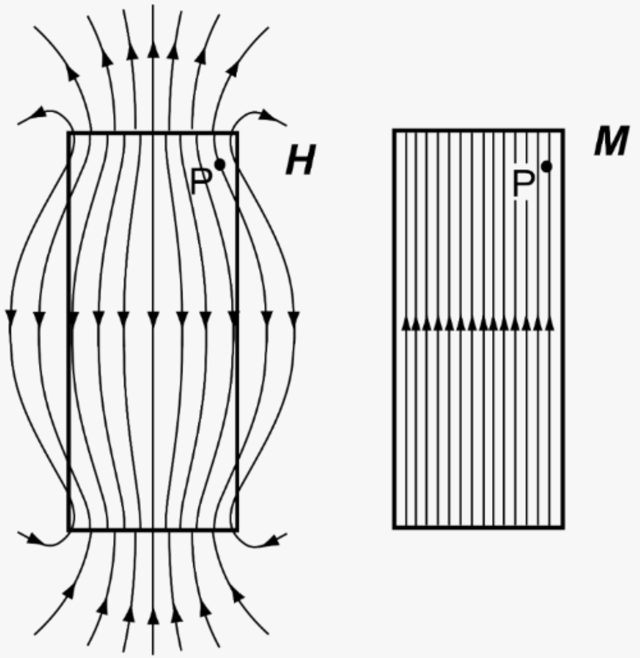
Example Permanent magnet (uniformly-magnetized)



- Surface charges

$$\sigma(\mathbf{r})=M_s\mathbf{m}(\mathbf{r})\cdot\mathbf{n}(\mathbf{r})$$

- Dipolar field

$$\mathbf{H}_\mathrm{d}(\mathbf{r})=\oiint\frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

Illustration from: M. Coey's book

Vocabulary

- Generic names
  - Magnetostatic field
  - Dipolar field
- Inside material
  - Demagnetizing field
- Outside material
  - Stray field

B versus H – Amperian versus Coulombian – Continuity conditions

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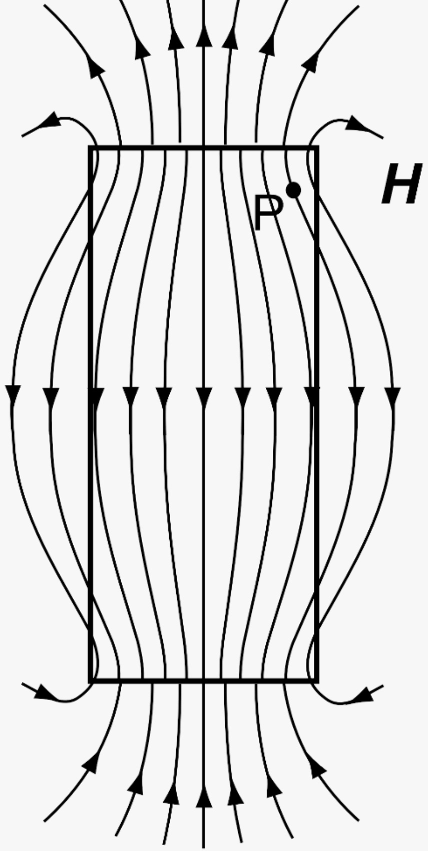
Coulombian description

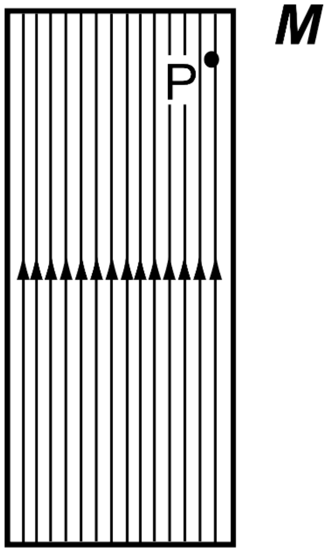
- Pseudo-charges source of H_d



$\nabla \times \mathbf{H} = \mathbf{0}$

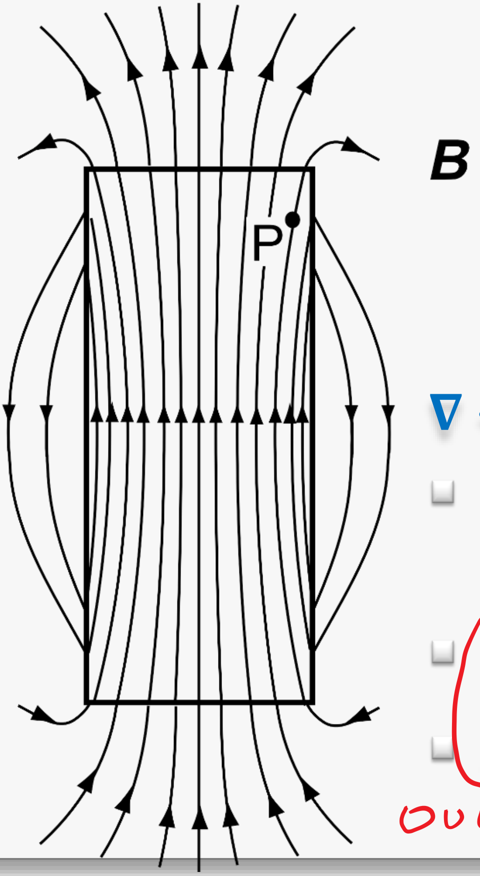
- No closed lines
- $\Delta H_{\parallel} = 0$
- $\Delta \mathbf{H} \cdot \mathbf{n} = \sigma$

out - in



Amperian description

- Fictitious currents source of B



$\nabla \cdot \mathbf{B} = \mathbf{0}$

- No magnetic monopole
- $\Delta B_{\perp} = 0$
- $\Delta \mathbf{B} = \mu_0 \mathbf{j} \times \mathbf{n}$

out - in

From: M. Coey's book

Dipolar energy and demagnetizing tensor

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Size considerations

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \mathrm{Volume} + \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{S}'$$

- Unchanged if all lengths are scaled: homothetic. NB: the following is a solid angle:

$$\mathrm{d}\mathbf{\Omega} = \frac{(\mathbf{r}-\mathbf{r}')\mathrm{d}\mathcal{S}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

- $\mathrm{H_d}$ does not depend on the size of the body
- Said to be a long-range interaction

Dipolar energy

- Zeeman energy of microscopic volume

$$\delta\mathcal{E}_\mathrm{Z} = -\mu_0 \mathbf{M}\delta\mathcal{V}\cdot\mathbf{H}_\mathrm{ext}$$

- Elementary volume of a macroscopic system creating its own dipolar field

$$E_\mathrm{d} = \delta\mathcal{E}_\mathrm{d}/\delta\mathcal{V} = -\frac{1}{2}\mu_0\mathbf{M}\cdot\mathbf{H}_\mathrm{d}$$

mutual energy

- Total dipolar energy of macroscopic body

$$\mathcal{E}_\mathrm{d} = -\frac{1}{2}\mu_0 \iiint_\mathcal{V} \mathbf{M}\cdot\mathbf{H}_\mathrm{d}\,\mathrm{d}\mathcal{V}$$

$$\mathcal{E}_\mathrm{d} = \frac{1}{2}\mu_0 \iiint_\mathcal{V} \mathbf{H}_\mathrm{d}^2\,\mathrm{d}\mathcal{V}$$

- Always positive. Zero means minimum

Demagnetizing coefficients – Maths

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Dipolar energy for uniform magnetization

$$\mathbf{M}(\mathbf{r}) = \mathbf{M} = M_\mathrm{s}(m_x\hat{\mathbf{x}} + m_y\hat{\mathbf{y}} + m_z\hat{\mathbf{z}})$$

- No volume charges: $\rho(\mathbf{r}) = -M_\mathrm{s}\,\boldsymbol{\nabla}\cdot\mathbf{m}(\mathbf{r}) = 0$

Implicit $\sum_{i=x,y,z}$

- Dipolar field: $$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \oiint_{\partial\mathcal{V}} \frac{[\mathbf{M}(\mathbf{r}')\cdot\mathbf{n}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}' = M_\mathrm{s}m_i \oiint_{\partial\mathcal{V}} \frac{\mathrm{n}_i(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$
- Dipolar energy:

$$\mathcal{E}_\mathrm{d} = -\frac{1}{2}\mu_0 \iiint_{\mathcal{V}} \mathbf{M}(\mathbf{r})\cdot\mathbf{H}_\mathrm{d}(\mathbf{r})\mathrm{d}\mathcal{V} = -\frac{1}{2}\mu_0 M_\mathrm{s}^2 m_i \iiint_{\mathcal{V}} \mathrm{d}\mathcal{V} \oiint_{\partial\mathcal{V}} \frac{n_i(\mathbf{r}')\,\mathbf{m}\cdot(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

$$\mathcal{E}_\mathrm{d} = -K_\mathrm{d} m_i m_j \iiint_{\mathcal{V}} \mathrm{d}\mathcal{V} \oiint_{\partial\mathcal{V}} \frac{n_i(\mathbf{r}')\,(r_j - r_j')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

Implicit $\sum_i \sum_{j=x,y,z}$

$$\langle\mathbf{H}_\mathrm{d}(\mathbf{r})\rangle = -M_\mathrm{s}\,\bar{\bar{\mathbf{N}}}\cdot\mathbf{m}$$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d}V\,\mathbf{m}\cdot\bar{\bar{\mathbf{N}}}\cdot\mathbf{m}$$

See more detailed approach: M. Beleggia et al., JMMM 263, L1-9 (2003)

Demagnetizing coefficients – Take-away messages

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For any shape of body

$$\langle \mathbf{H}_\mathrm{d}(\mathbf{r}) \rangle = -M_s \, \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V \, \mathbf{m} \cdot \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

Dipolar anisotropy is always of second order

- $\bar{\bar{\mathbf{N}}}$ demagnetizing tensor. Always positive, and can be diagonalized. $N_x + N_y + N_z = 1$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V \left(N_x m_x^2 + N_y m_y^2 + N_z m_z^2 \right)$$

- Along main directions

$$\langle H_{\mathrm{d},i}(\mathbf{r}) \rangle = -N_i M_s$$

Hypothesis uniform **M** may be too strong
Remember: dipolar field is NOT uniform

For ellipsoids etc.

- Condition: boundary is a polynomial of the coordinates, with degree at most two

Slabs (thin films), cylinders, ellipsoids

$$z^2 = \left(\frac{t}{2}\right)^2 \qquad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \qquad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

$$\mathbf{H}_\mathbf{d} = -M_s \, \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

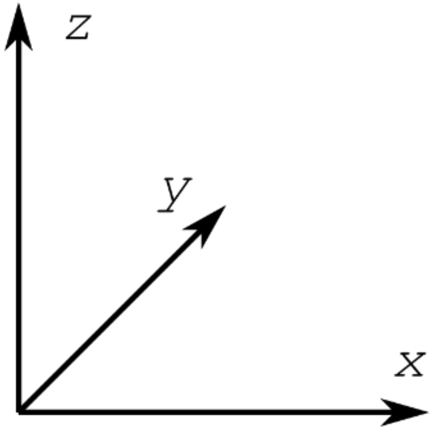
$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V \, \mathbf{m} \cdot \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

- Along main directions

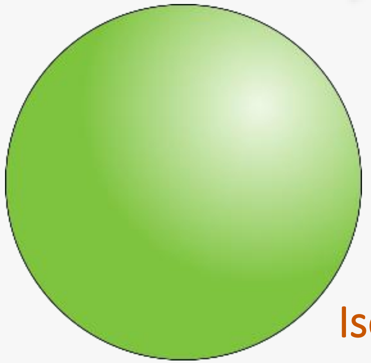
$$H_{\mathrm{d},i} = -N_i M_s$$

M and **H** may not be colinear along non-main directions

Demagnetizing coefficient (examples)



Sphere $L_x = L_y = L_z = D$

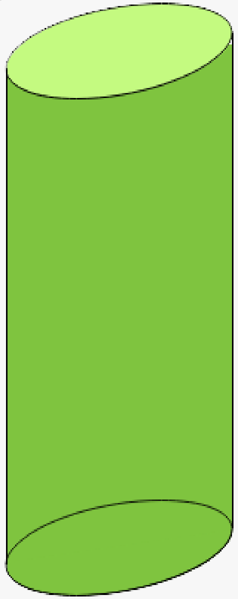


$$N_x = N_y = N_z = \frac{1}{3}$$

Isotropic

Cylinder $L_x = L_y = D$

$L_z = \infty$



$$N_x = N_y = \frac{1}{2}$$

$$N_z = 0$$

Favors **axial magnetization**

Slab (thin film) $L_x = L_y = \infty$

$$N_x = N_y = 0$$

$$N_z = 1$$

Favors in-plane magnetization

Take-away message

Dipolar energy favors alignment of magnetization with longest direction of sample

The F_{ijk} functions

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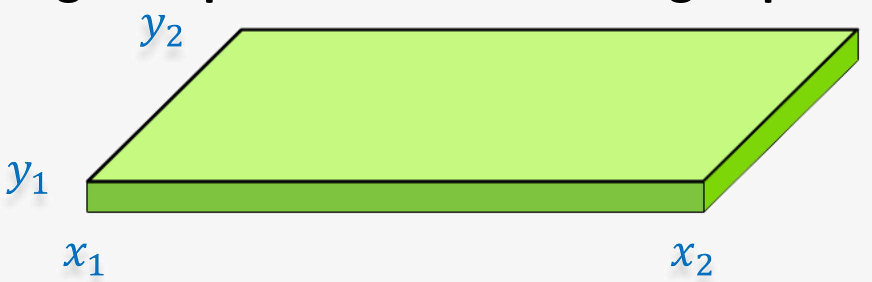
Core function for the magnetic scalar potential

$$F_{000}(x,y,z)=\frac{1}{r}=\frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$\phi(x,y,z)=\frac{Q}{4\pi}F_{000}(x,y,z)$$

Magnetic potential from a charged plate



$$\phi(x,y,z)=\frac{\sigma}{4\pi}\int_{x_1}^{x_2}\mathrm{d}x'\int_{y_1}^{y_2}\mathrm{d}y'\,F_{000}(x-x',y-y',z)$$

Sum of four terms involving here F_{110}

Definition for F_{ijk} function

F_{000} Can be integrated and/or derived analytically to any order, against x, y, z

$F_{ijk}(x,y,z)$

- Integrated *i* times versus *x*
- Integrated *j* times versus *y*
- Integrated *k* times versus *z*

Example

$$F_{100}(x,y,z)=\int F_{000}(x,y,z)\mathrm{d}x+Cste$$

A. Hubert, R. Schäfer, Magnetic domains, Springer (2000)

The H_{ijk} functions – Examples of use (for an *xy* charged plate)

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Magnetic scalar potential

F_{110} Magnetic scalar potential

Components of magnetic field

F_{11-1} $H_{d,z}$

F_{010} $H_{d,x}$

F_{100} $H_{d,y}$

Gradients of magnetic field

F_{11-2} $\dfrac{\mathrm{d}H_{d,z}}{\mathrm{d}z}$

F_{01-1} $\dfrac{\mathrm{d}H_{d,x}}{\mathrm{d}z}$

F_{10-1} $\dfrac{\mathrm{d}H_{d,y}}{\mathrm{d}z}$

MFM contrast

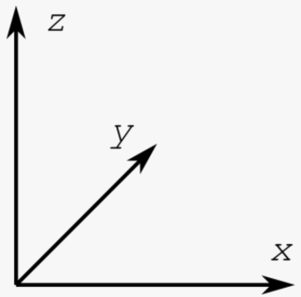
Demag coefficients

F_{220} N_z

F_{022} N_x

F_{202} N_y

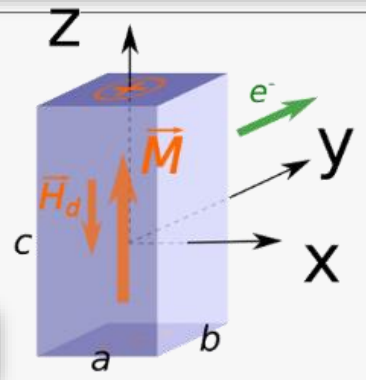
Examples of other specific cases



F_{12-1} *z* component of H, integrated along *y*

F_{120} *z*-averaged *z* component of H, integrated along *y*

Electron holography of a PSA



Range considerations

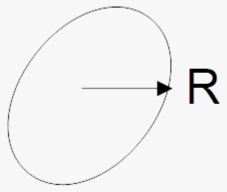
Range

Example: upper bound of dipolar field in thin films

Integration

$$\|\mathbf{H}_\mathrm{d}(\mathbf{r})\| \leq M_\mathrm{s} t \int \frac{2\pi r}{r^3} \mathrm{d}r$$

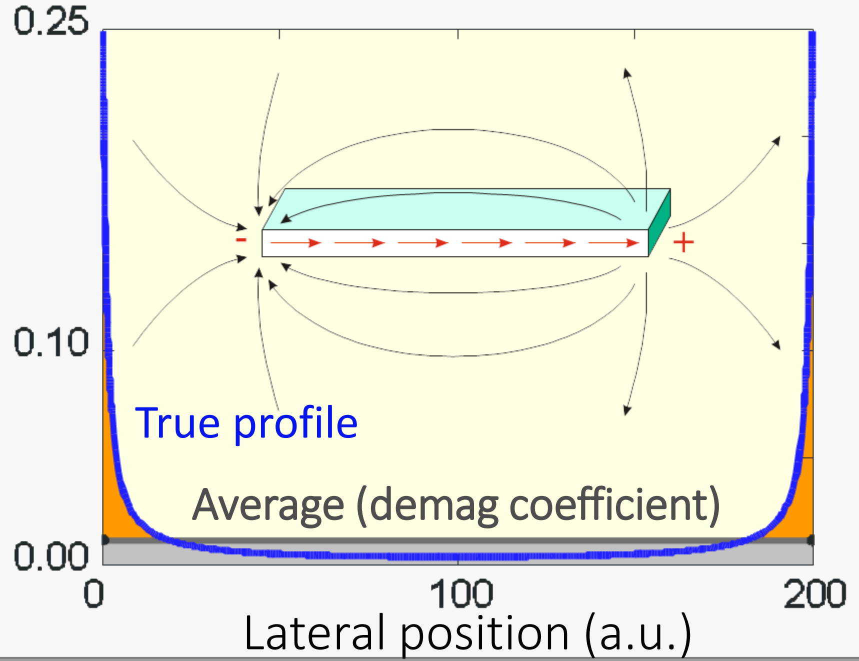
Hd for dipole



$$\Rightarrow \quad \|\mathbf{H}_\mathrm{d}(\mathbf{R})\| \leq \mathrm{Cste} + \mathcal{O}(1/R)$$

Non-homogeneity

Example: flat strip with aspect ratio 0.0125



- Dipolar fields are short-ranged and inhomogeneous in low dimensions
- Consequences: non-uniform magnetization switching, edge modes etc.

⇒ A 1D/2D system in space behaves very differently from a nano-bulk magnet

Units in Magnetism

Definitions

	S.I.		cgs-Gauss	
	Meter	m	Centimeter	cm
	Kilogram	kg	Gram	g
	Second	s	Second	s
	Ampere	A	Ab-Ampere	ab-A = 10 A

$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ | $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

$\mu_0 = 4\pi \times 10^{-7}$ S.I. | $"\mu_0" = 4\pi.$

Problems with cgs

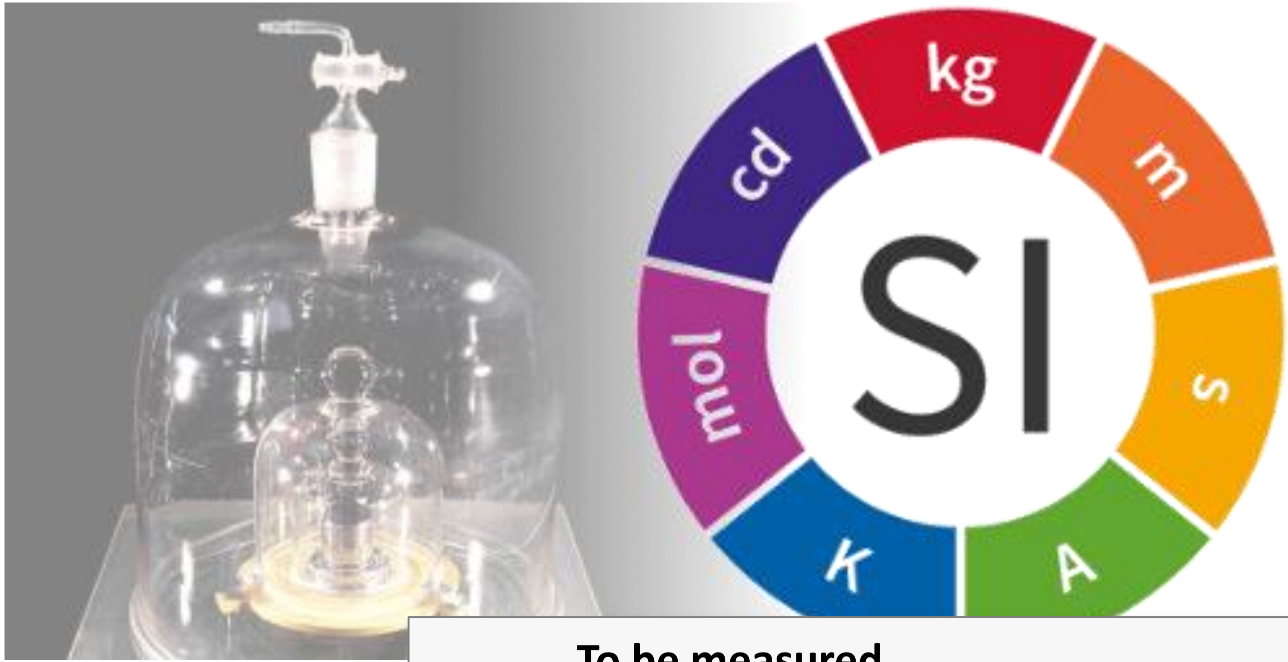
- The quantity for charge current is missing
  No check for homogeneity;
  paradox for spintronics
- Inconsistent definition of H
  Dimensionless quantities are affected:
  demag coefficients, susceptibility etc.

Conversion

Field	$\mathbf{H}$	$1\ \mathrm{A/m}$	⟷	$4\pi \times 10^{-3}\ \mathrm{Oe}$	Œrsted
Moment	$\boldsymbol{\mu}$	$1\ \mathrm{A \cdot m^2}$	⟷	$10^3\ \mathrm{emu}$	
Magnetization	$\mathbf{M}$	$1\ \mathrm{A/m}$	⟷	$10^{-3}\ \mathrm{emu/cm^3}$	Electromagnetic Unit
Induction	$\mathbf{B}$	$1\ \mathrm{T}$	⟷	$10^4\ \mathrm{G}$	Gauss
Susceptibility	$\chi = M/H$	1	⟷	$1/4\pi$	

Tutorial on units
Questions: http://magnetism.eu/esm/2018/abs/fruchart-practical-abs1.pdf
Answers: http://magnetism.eu/esm/2018/abs/fruchart-practical-answers1.pdf

Quantum revolution in SI units in 2019



Define quantities

- Times
- Length
- Mass
- Electric charge

Fixed values

- Speed of light -> Define meter
- Planck constant -> Defines kg
- Charge of the electron

To be measured

- Magnetic permeability of vacuum

$$\mu_0 \neq 4\pi \times 10^{-7}\ \mathrm{S.I.}$$

$$\mu_0 = 4\pi[1 + 2.0(2.3) \cdot 10^{-10}] \times 10^{-7}\ \mathrm{S.I.}$$

R. B. Goldfarb, IEEE Trans. Magn. MAG. 8, 1-3 (2017); R. B. Goldfarb, IEEE Mag. Lett. 9, 1205905 (2018)
S. Schlamminger, Redefining the kilogram and other SI units, IOP Physics World Discovery (2018)

What is dimensionality?

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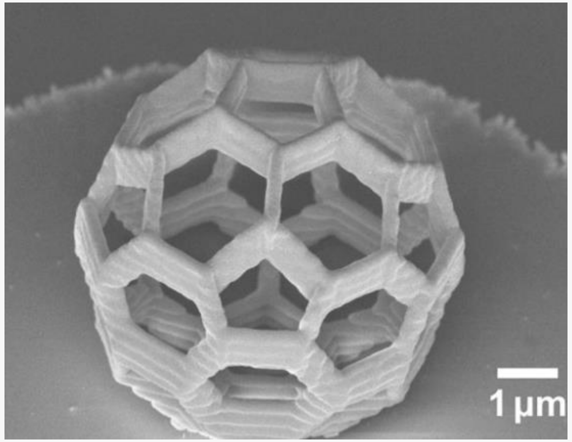
Space

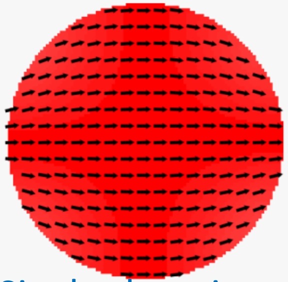
- Physical quantities (incl. magnetization) defined at any location in space

$$\mathbf{M} = \mathbf{M}(x, y, z)$$

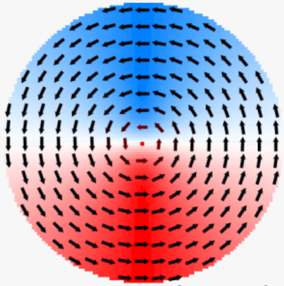


C. Donnnelly, Phys. Rev. Lett. 114, 115501 (2015)

- Features: shape, dimensions, dimensionality



Single-domain

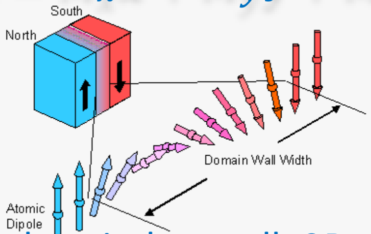


Size > magn. length scale

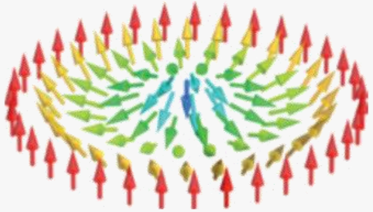
Magnetization components

- Vector field for magnetization has three components

$$\mathbf{M} = M_x\hat{\mathbf{x}} + M_y\hat{\mathbf{y}} + M_z\hat{\mathbf{z}}$$



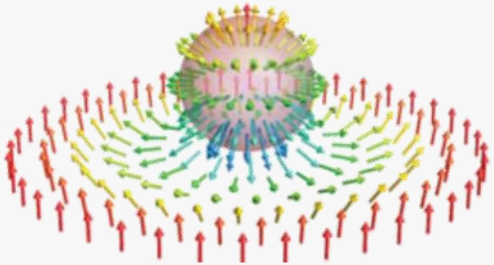
Simple spiral or wall: 2D



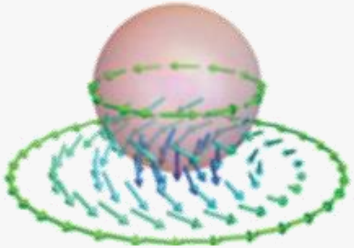
A. Fert, Nat. Nanotech. 8, 152 (2013)

Skyrmion: 3D

- Mapping magnetization on the unit sphere



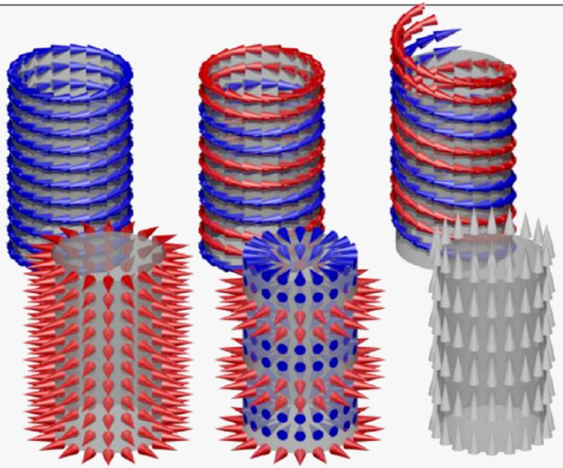
Skyrmion



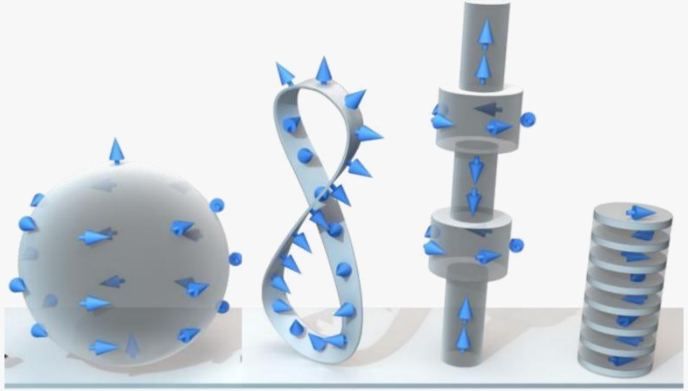
Vortex

In: H.B. Braun, *Solitons in real space: domain walls, vortices, hedgehogs and skyrmions*, Springer (2018)

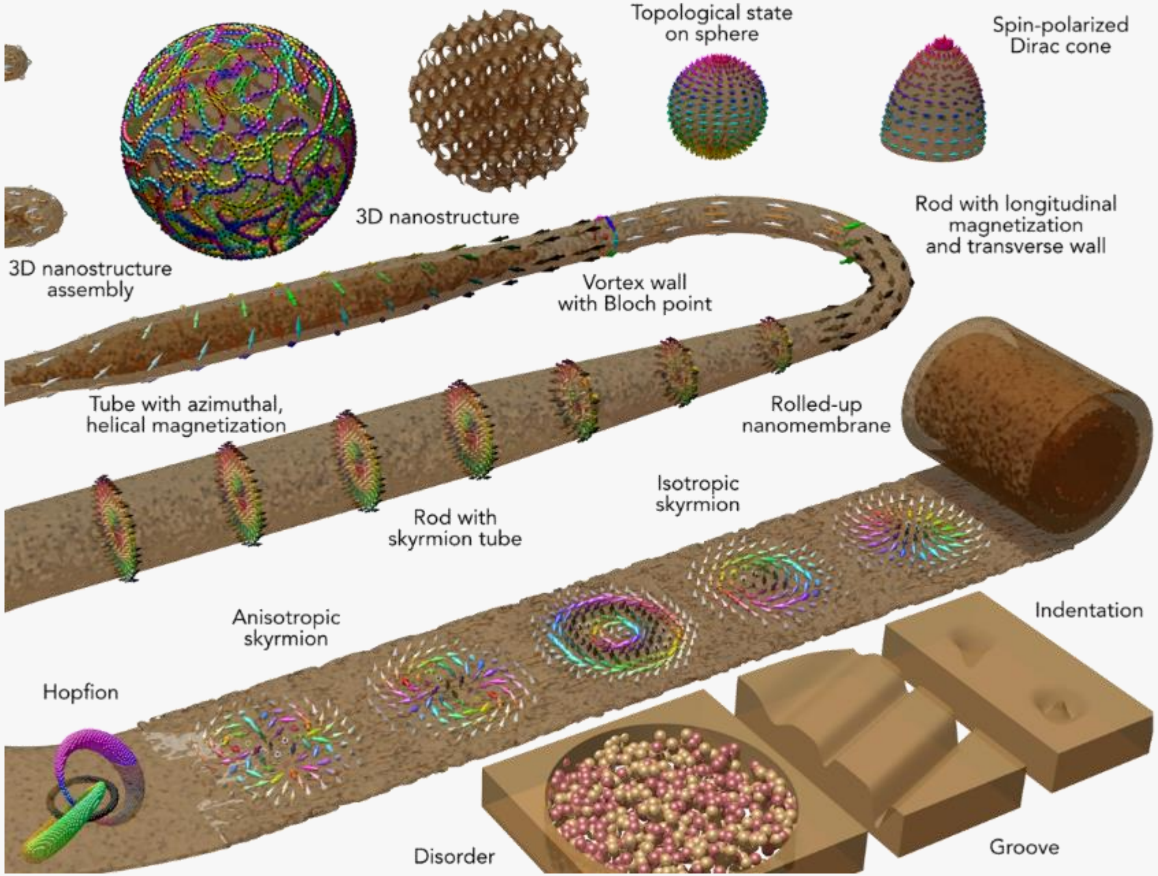
Sharp rise of contributions on 3D nanomagnetism



R. Streubel, J.Phys.D: Appl.Phys. 49, 363001 (2016)



A. Fernandez-Pacheco, Nat. Comm., 8, (2017)



A. Fernandez-Pacheco, Nat. Comm., 8, (2017)

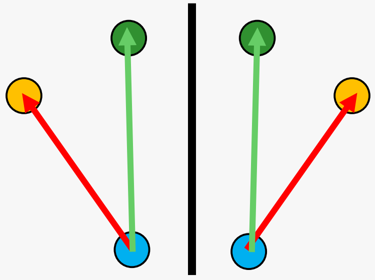
Lecture Denys Makarov

Symmetry and chirality

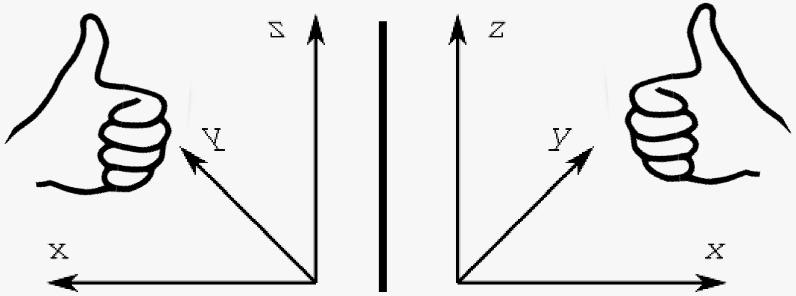
Definition

An object that cannot be superimposed onto itself, following a mirror symmetry

- Two vectors do not allow chirality (the image can be flipped 180°)

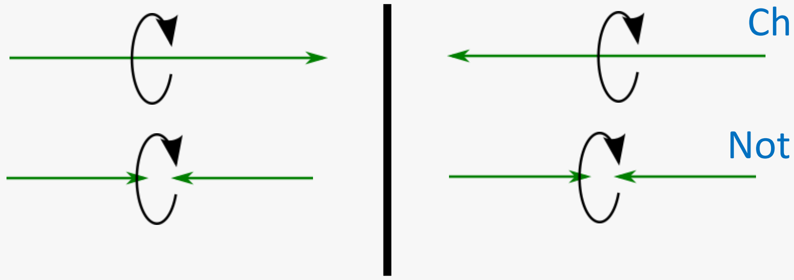


- Three vectors are required for chirality



(Counter-)examples

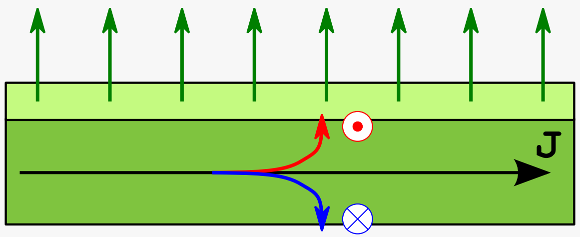
- Not all curling structures are chiral



- Competition or promotion with chiral physical effects, i.e., those involving a vector product

LLG equation

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -|\gamma_0|\mathbf{m} \times \mathbf{H} + \alpha\mathbf{m} \times \frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t}$$

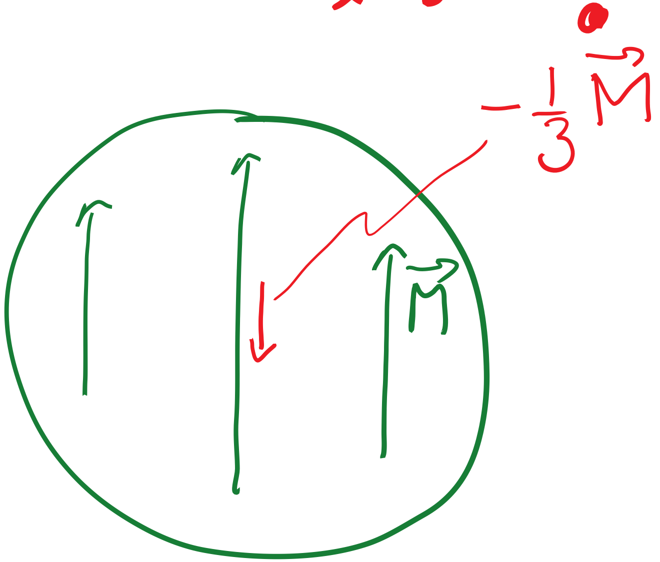


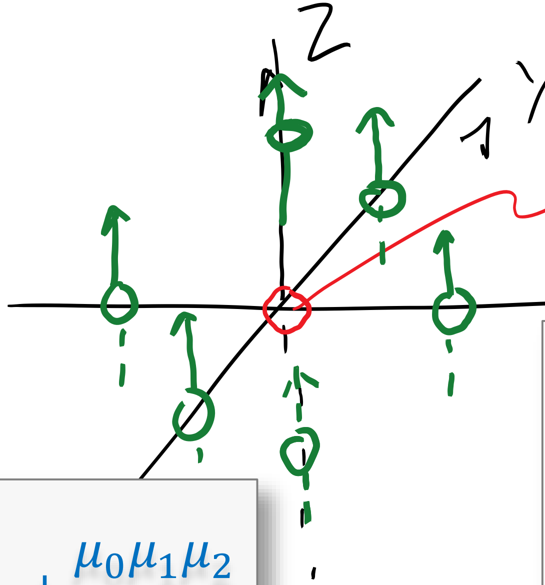
Spin-orbit torques

Lecture Hyunsoo Yang

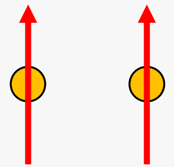
Quizz #1

I can prove that demagnetizing field does NOT exist !





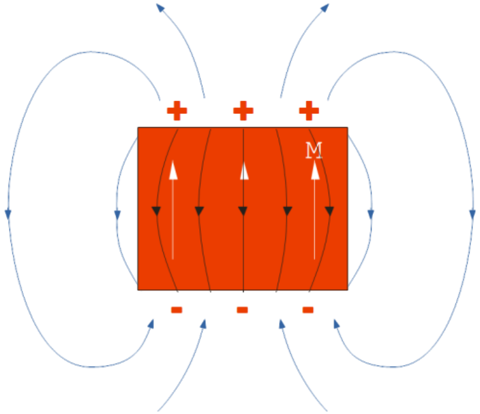
$\vec{H_d} = \vec{0}$

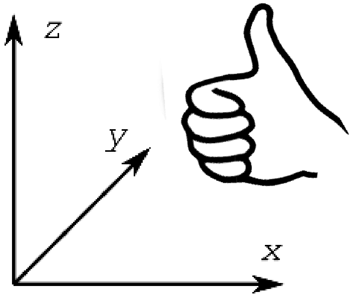


$$\mathcal{E} = +\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

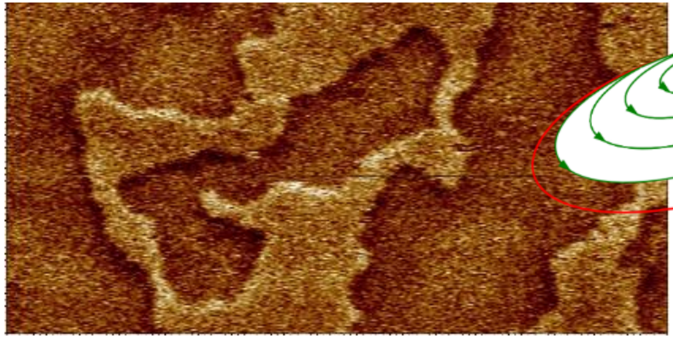
$$\mathcal{E} = -2\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

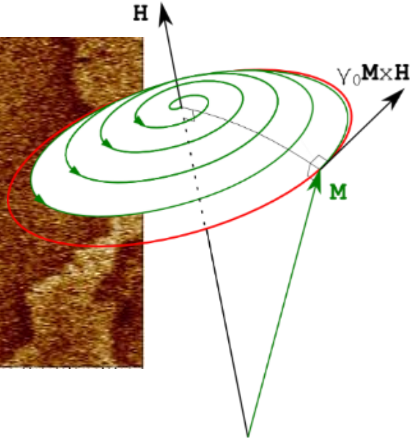
- Units and fields



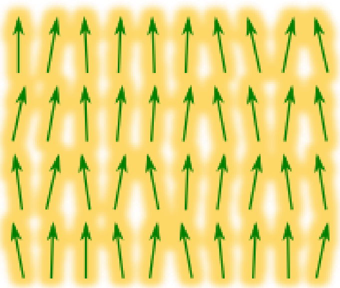


- Magnetizaiton processes and micromagnetism basics





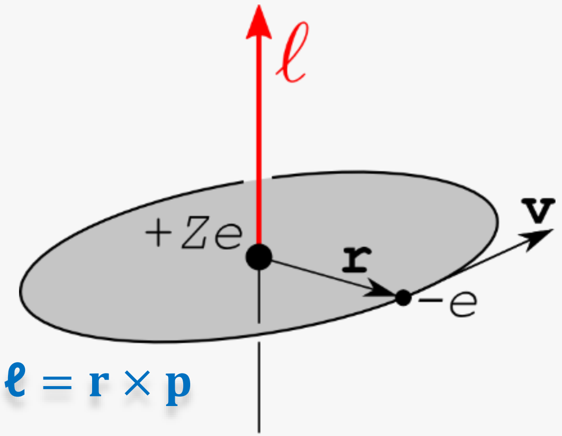
- Magnetic ordering and materials



Moments

Angular momentum

Classical view: electron orbiting around the nucleus



$\boldsymbol{\ell} = \mathbf{r} \times \mathbf{p}$

$\ell = m_e r v$

Niels Bohr postulate: is quantized

$$\ell = m_e r v \in \hbar\mathbb{N}$$

$$\hbar = \frac{h}{2\pi} = 1.0546 \times 10^{-34}\ \mathrm{J \cdot s}$$

Orbital magnetic moment

Results from angular momentum

$$\boldsymbol{\mu} = \frac{1}{2}\iiint \mathbf{r} \times \mathbf{j}(\mathbf{r})\mathrm{d}\mathbf{r} = I\mathcal{S}$$

$$\mu = \pi r^2 I = -erv/2 \qquad \mathrm{A \cdot m^2}$$

Gyromagnetic ratio γ

Magnetic moment associated with angular momentum: $\boldsymbol{\mu} = \gamma\boldsymbol{\ell}$

For the orbital motion of electrons: $\gamma = -\dfrac{e}{2m_e}$

Bohr magneton μ_B

Quantum for magnetic moments, resulting from the quantization of angular momentum

$$\mu_B = \gamma\hbar \qquad \mu_B = 9.274 \times 10^{-24}\ \mathrm{A \cdot m^2}$$

Moments

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Spin magnetic moment

- Spin = intrinsically-quantized angular momentum
- Electrons are fermions (half-integer spin)

$$s = \pm \frac{1}{2}$$

- Angular momentum $s\hbar = \pm \frac{\hbar}{2}$
- Magnetic moment (Dirac equation, not classical)

$$\gamma = -\frac{e}{m_{\mathrm{e}}}$$

→ Electrons carry a spin magnetic moment $\approx 1\ \mu_{\mathrm{B}}$

Gyromagnetic ratio γ

- Magnetic moment associated with angular momentum $\boldsymbol{\mu} = \gamma \boldsymbol{\ell}$
- Orbital motion of electrons $\gamma \approx -\frac{e}{2m_{\mathrm{e}}}$
- Spin of electrons $\gamma \approx -\frac{e}{m_{\mathrm{e}}}$

Bohr magneton μ_{B}

- Quantum for magnetic moment, resulting from the quantization of angular momentum

$\mu_{\mathrm{B}} = \gamma\hbar$ $\qquad$ $\mu_{\mathrm{B}} = 9.274 \times 10^{-24}\ \mathrm{A \cdot m^2}$

Landé factor $\frac{|\boldsymbol{\mu}|}{\mu_{\mathrm{B}}} = g\frac{|\boldsymbol{\ell}|}{\hbar}$

- Orbital moment $g = 1$
- Electron spin $g \approx 2$

Magnetic exchange

Physics

- Spin + space wave function must be antisymmetric
- Coulomb repulsion
- Pauli exclusion

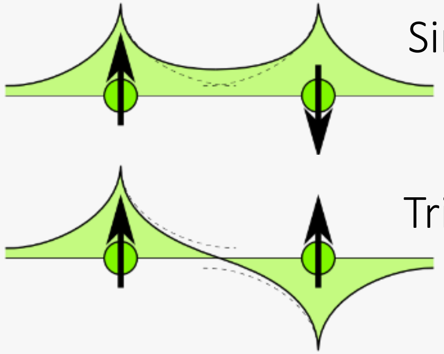
➔ May be viewed as interatomic Hund's rules

Hamiltonian

$$\mathcal{H} = -2J_{1,2}\mathbf{S}_1 \cdot \mathbf{S}_2$$

$J_{1,2}$ Exchange integral

Direct exchange



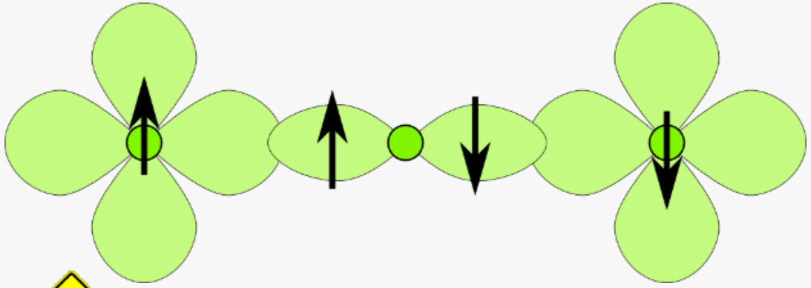
Singlet

Triplet

Molecules → Singlet

Metals ⟶ Ferro/Antiferro

Superexchange



Bond-length and – orientation dependent

Often: π ⟶ Antiferro; π /2 ⟶ ferro

Indirect exchange

Conduction electrons

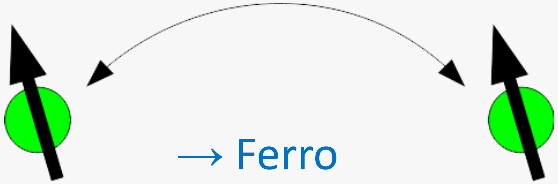
⟶ Ferro

RKKY, rare-earth (4f), GaMnAs (3d)

Double exchange

Mixed-valence states

e- hopping



⟶ Ferro

Mn^{3+} ($3d^4$) Mn^{4+} ($3d^3$)

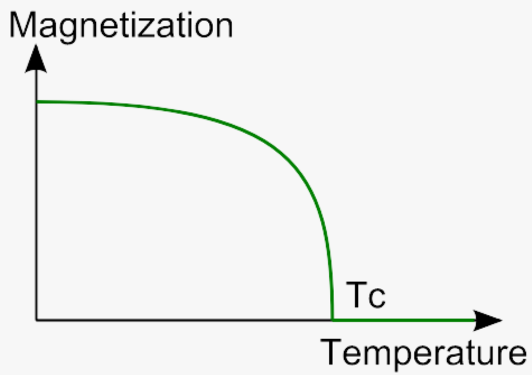
Example: $(La_{0.7}Ca_{0.3})MnO_3$

Magnetic ordering and orders

Magnetic ordering

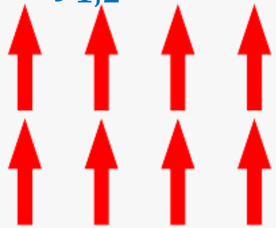
Magnetic exchange between microscopic moments:

$$\mathcal{E} = -2 \sum_{i<j} J_{1,2} \mathbf{S}_i \cdot \mathbf{S}_j$$



Ferromagnetism

$J_{1,2} > 0$



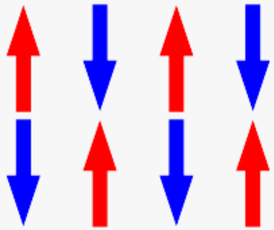
Fe

$T_C = 1043\ \mathrm{K}$

$M_s = 1.73 \times 10^6\ \mathrm{A/m}$

Antiferromagnetism

$J_{1,2} < 0$



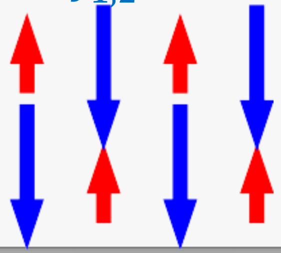
CoO

$T_N = 292\ \mathrm{K}$

$J = 3/2$

Ferrimagnetism

$J_{1,2} < 0$



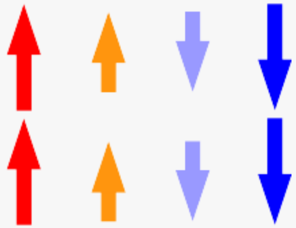
Fe_3O_4

$T_C = 858\ \mathrm{K}$

$M_s = 480\ \mathrm{kA/m}$

Helimagnetism

$J_1, J_2\ \dots$



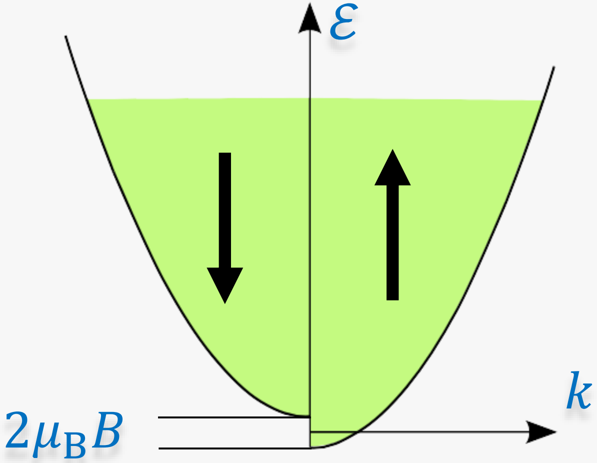
Dy

$T \in 85 - 179\ \mathrm{K}$

$\mu = 10.4\ \mu_B$

Magnetic exchange (band magnetism)

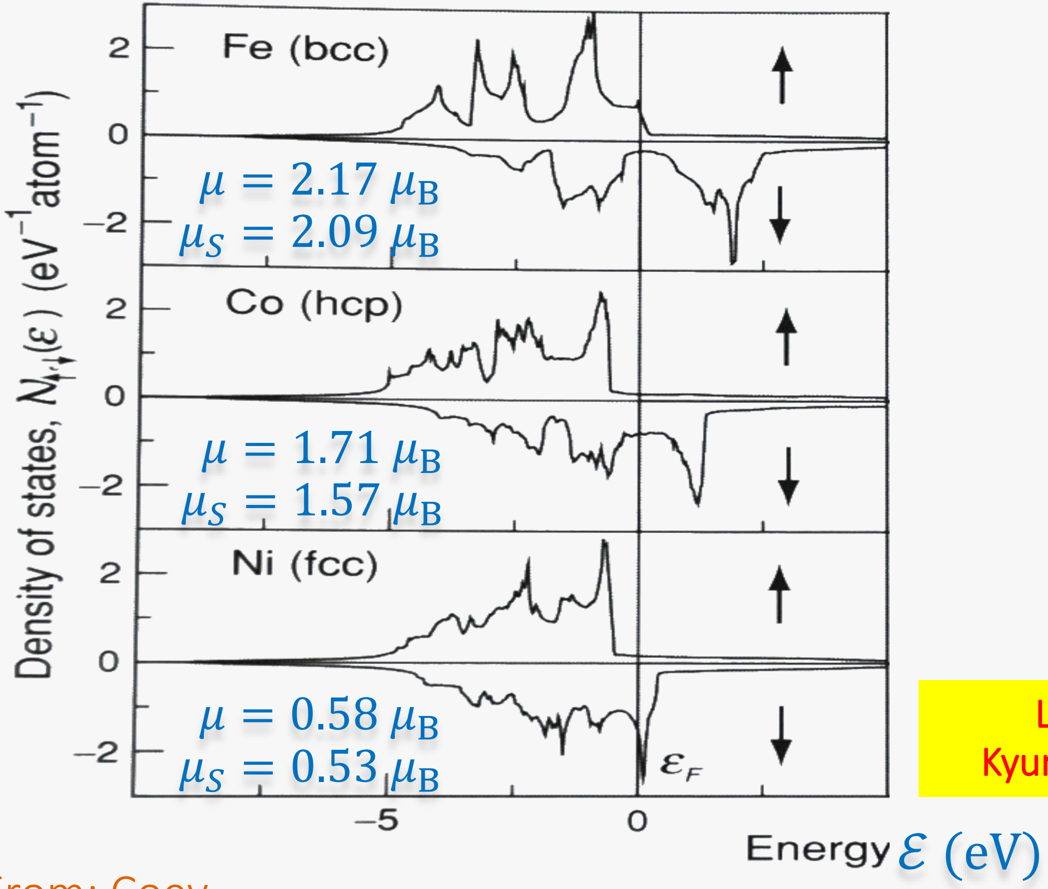
Stoner criterium



- Cost in kinetic energy
- Gain in Coulomb interaction

Criterion for ordering: $I\rho_{\uparrow,\downarrow}(\mathcal{E}_F)>1$

Spin-polarized band structure

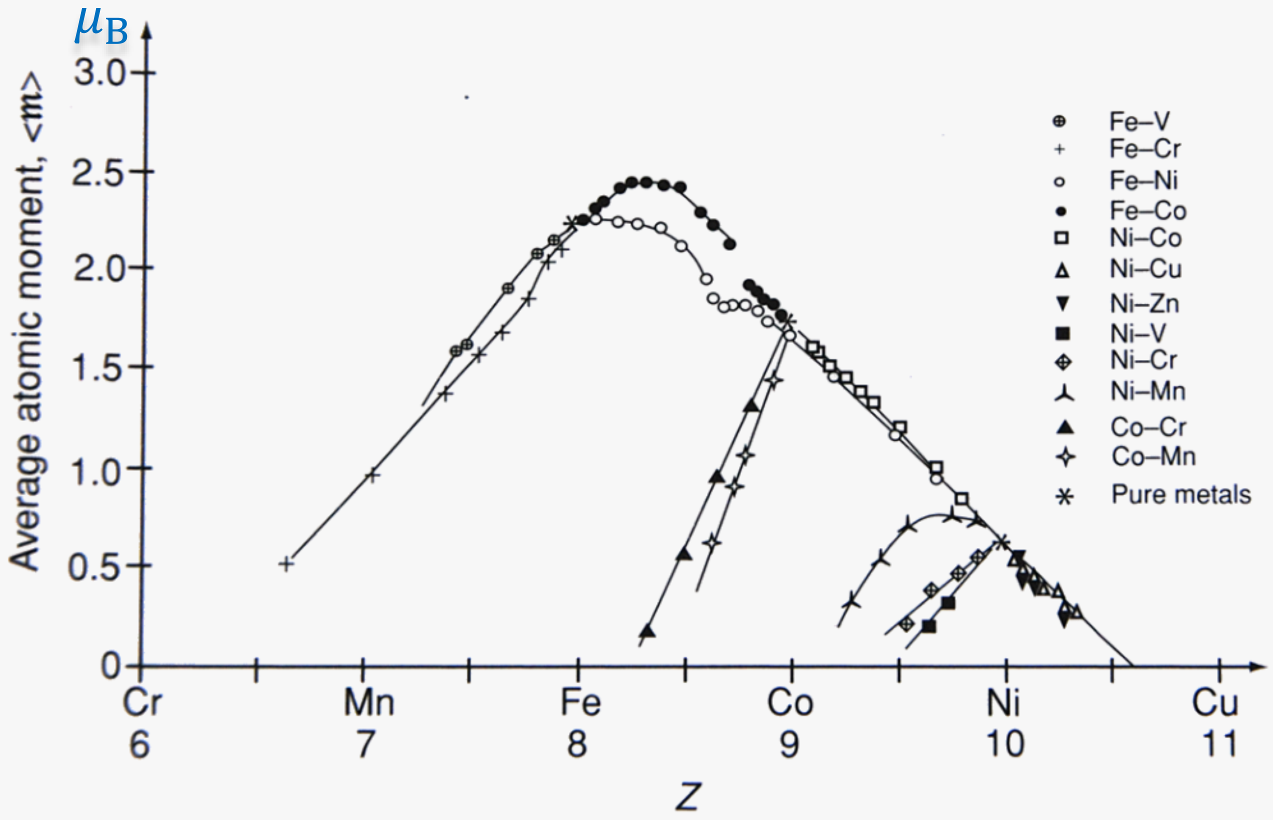


From: Coey

Lecture
Kyung-Jin Lee

Magnetic exchange (band magnetism)

Slater-Pauling curve Magnetic moment per atom versus 3d band filling



- Moment per atom is lower than for atomic species
- Reasonably well explained by a rigid flat band model
- Illustrates the transfer from 4s to 3d electrons

From: Coey

Magnetic ordering

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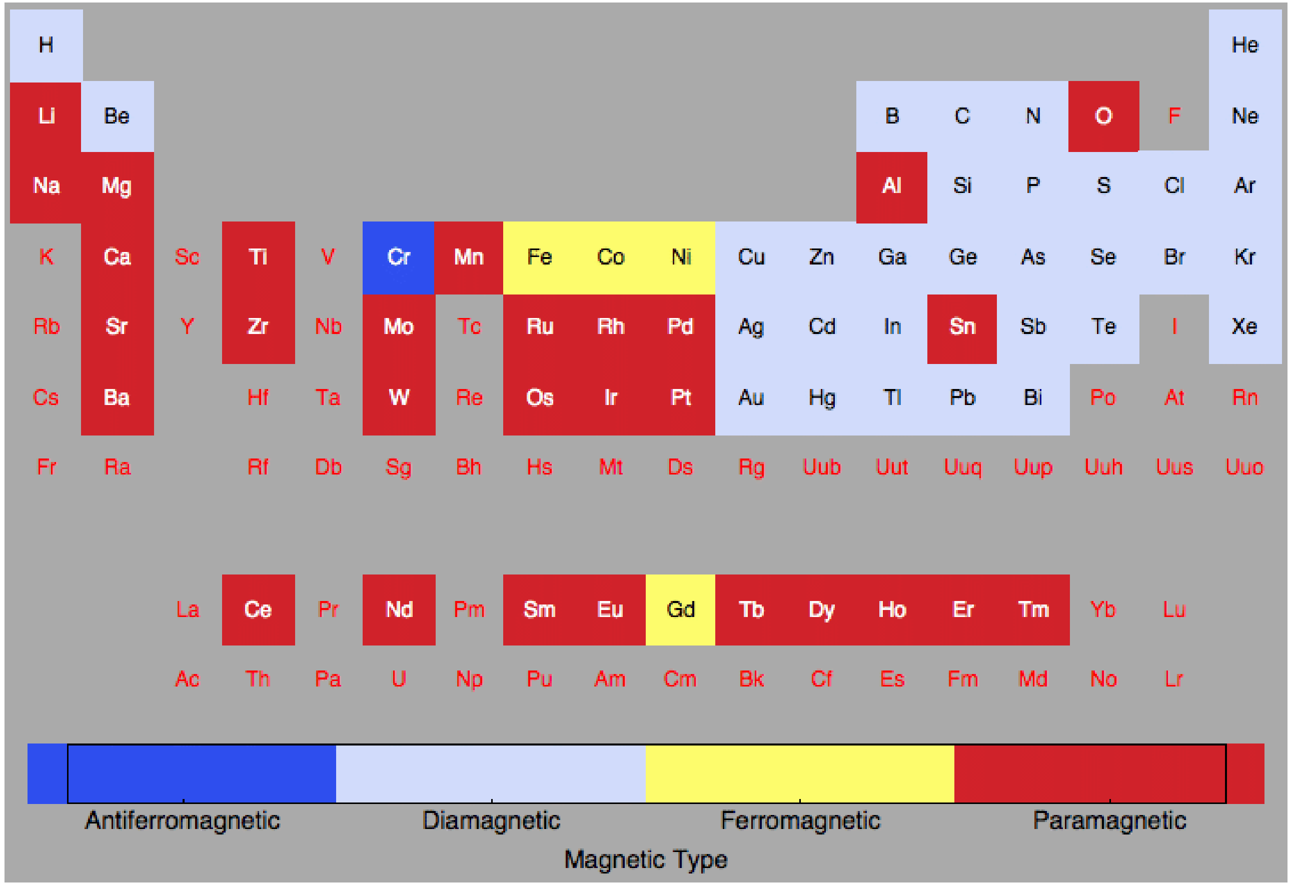
Magnetic properties at room temperature, single elements



periodic table.com

Ordering and dimensionality (theory)

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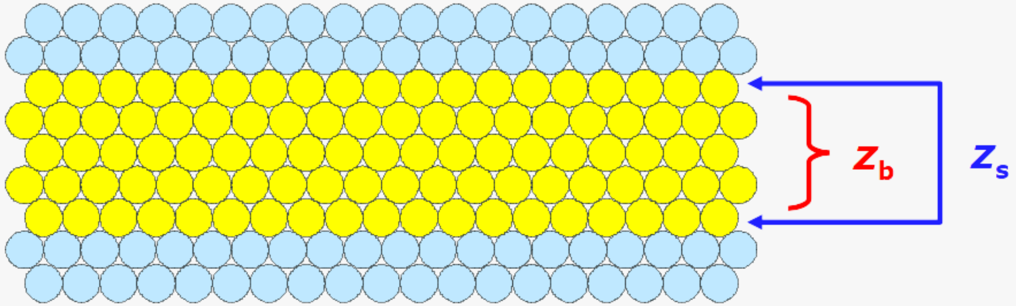
A bit of theory

- **Ising (1925)**. No magnetic order at T>0K in 1D Ising chain.
- **Bloch (1930)**. No magnetic order at T>0K in 2D Heisenberg (spin-waves)
  N. D. Mermin, H. Wagner, PRL17, 1133 (1966)
- **Onsager (1944) + Yang (1951)**. 2D Ising model: Tc>0K

→ Magnetic anisotropy promotes ordering

Lecture
Peng Li

Naïve views: mean field



$$T_C = \frac{\mu_0 z n_{W,1} n g_J^2 \mu_B^2 J(J+1)}{3k_B}$$

N atomic layers → $\langle z \rangle = z_b - \frac{2(z_b - z_s)}{N}$

nearest neighbors

→ $\Delta T_C(t) \sim 1/t$

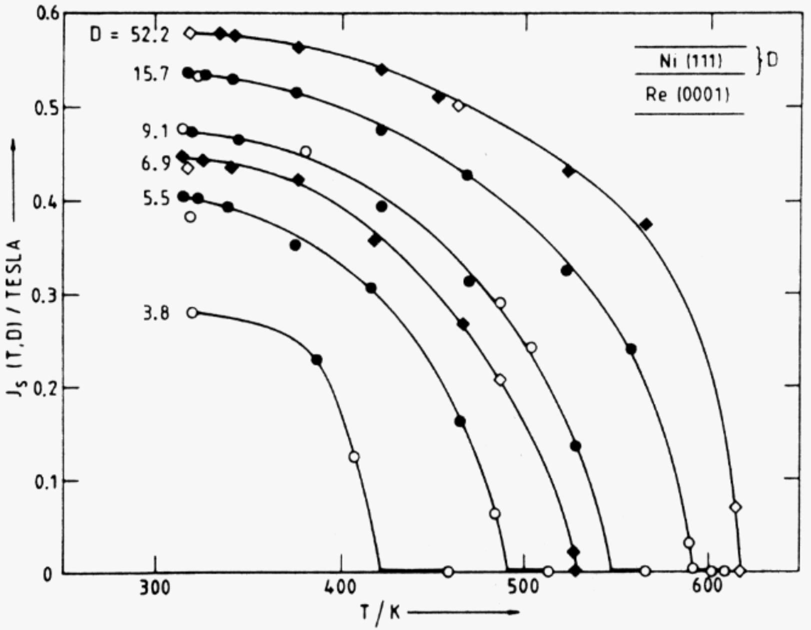
Confirmed by a more robust layer-dependent mean-field theory

G.A.T. Allan, PRB1, 352 (1970)

Ordering and dimensionality (experiments)

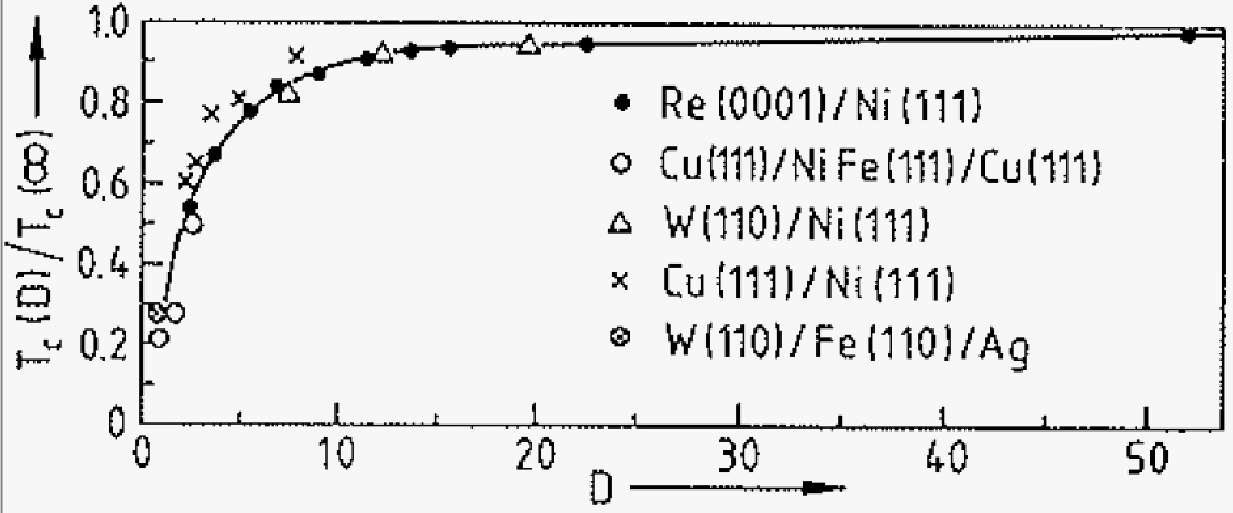
Qualitative

Ni(111)/Re(0001)



R. Bergholz and U. Gradmann, J. Magn. Magn. Mater. 45, 389 (1984)

Quantitative, master curve

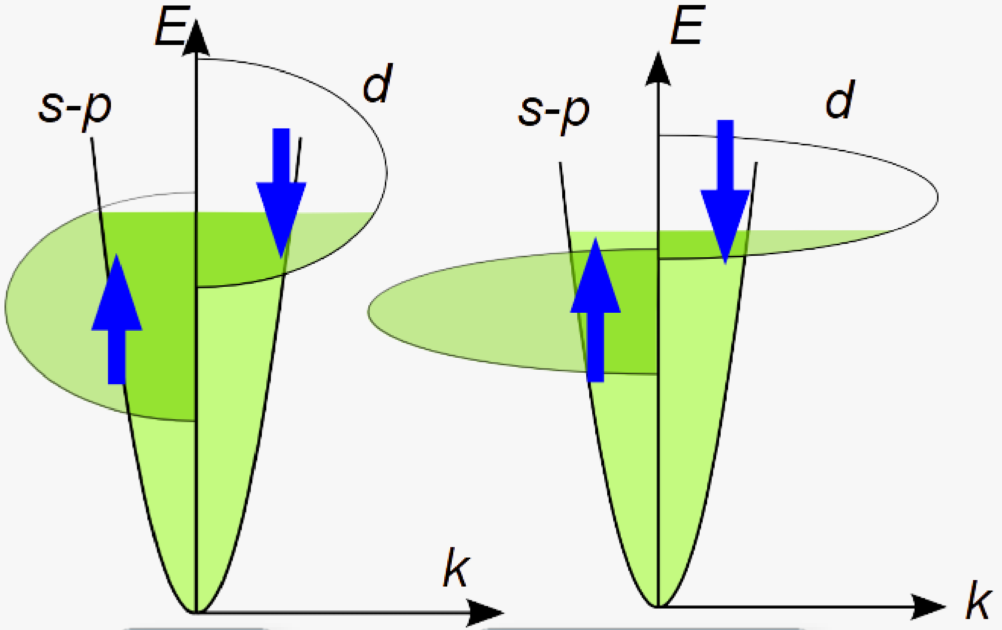


- Curie temperature well fitted by molecular field $\Delta T_C(t) \sim 1/t$

- Ordering temperature decreases with thickness
- Very significant below ≈ 1nm
- Check also: critical exponents

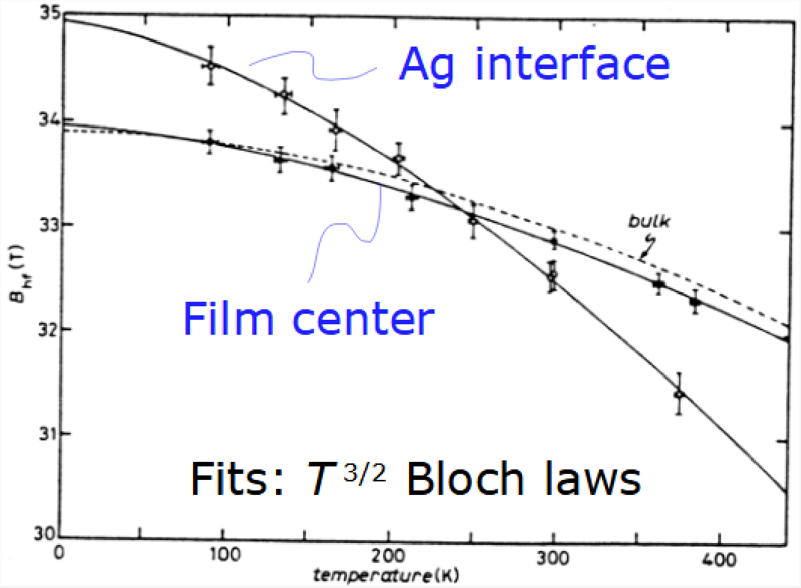
Magnetic moment versus dimensionality

Simple picture: band narrowing at surfaces



In practice

Ag/Fe(110)/W(110)

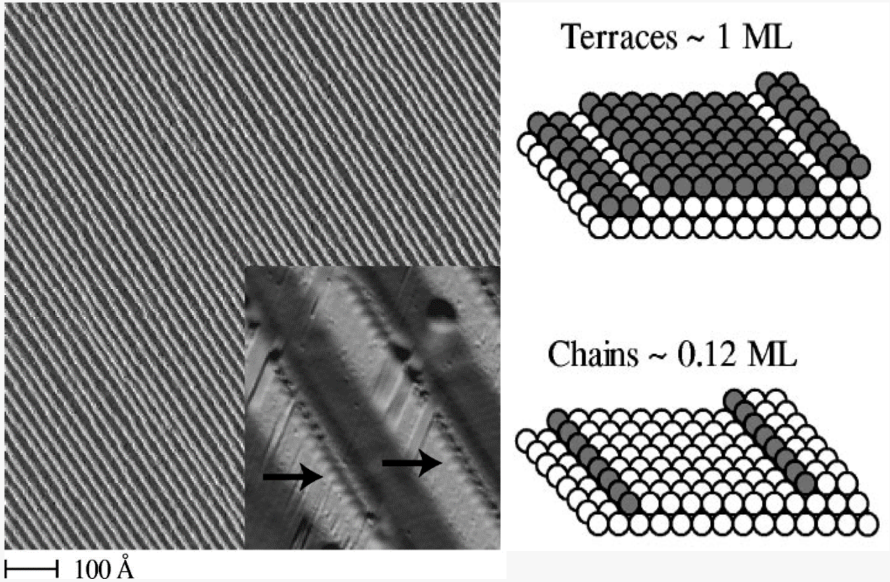


U. Gradmann et. al.

- Surface moments are usually 20-30% larger than in the bulk
- However, decay faster with temperature

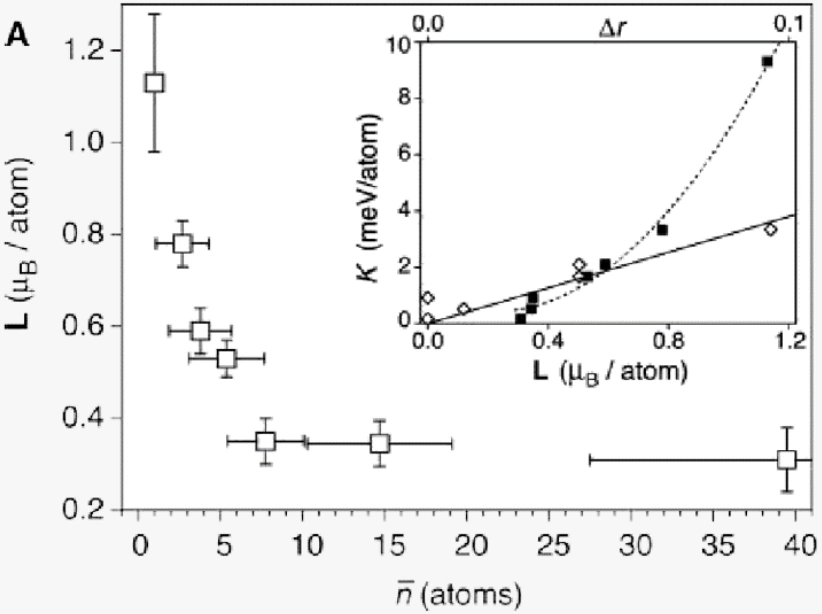
Magnetic moment versus dimensionality

Example of system: sub-monolayer Co/Pt(997)



A. Dallmeyer et al., Phys.Rev.B 61(8), R5153 (2000)

Example of system: Co/Pt(111)



P. Gambardella et al., Science 300, 1130 (2003)

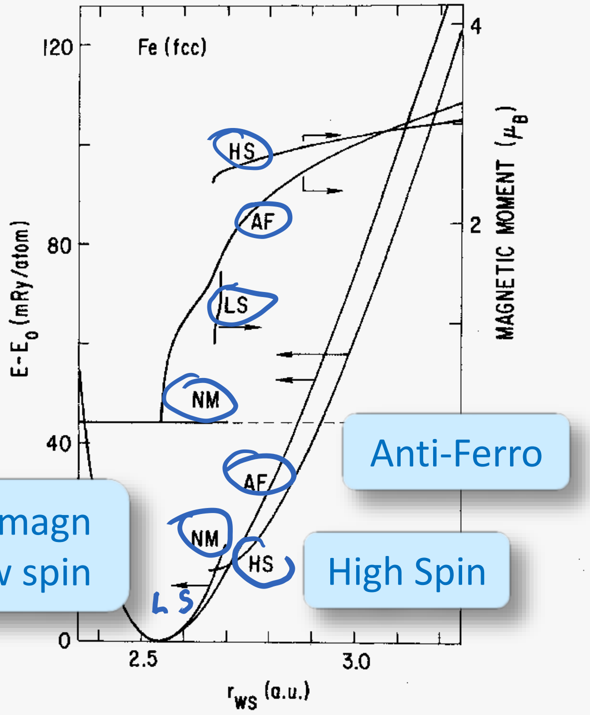
Conclusions

- From bulk to atoms: considerable increase of orbital moment
- 2 atoms closer to wire than 1 atom
- bi-atomic wire closer to surface than wire

Magnetic order versus dimensionality – Example: Fe

Theory (bulk)

- fcc γ-Fe for T>1185K: non-magnetic
- 'ground-state': sensitive on strain

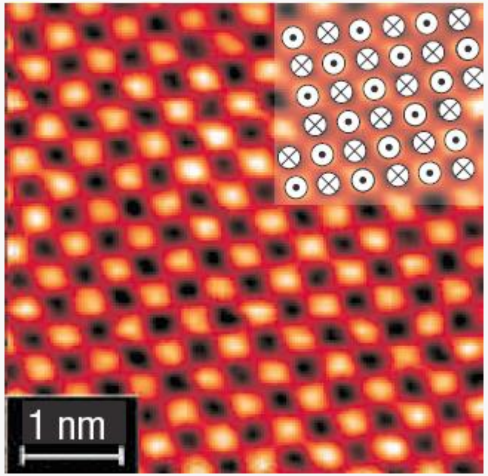


V. L. Moruzzi et al., PRB39, 6957 (1989)

See also: O.K. Andersen, Physica B 86, 249 (1977)

(Some) experiments in low dimensions

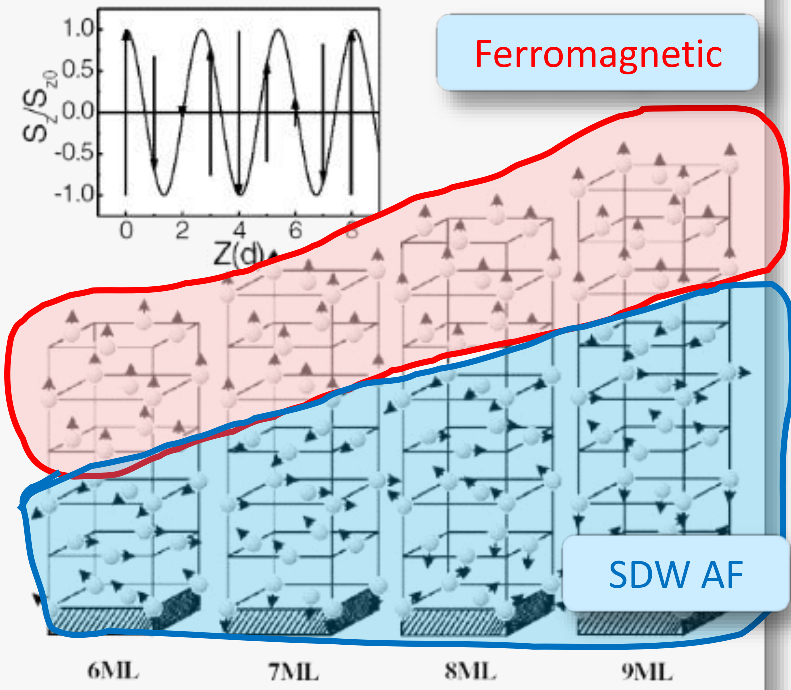
AF Fe(1ML)/W(001)



Antiferromagnetic domain (SP-STM)

M. Bode et al., Nat. Mater. 5, 477-481 (2006)

Spin-density-wave AF Fe(001)

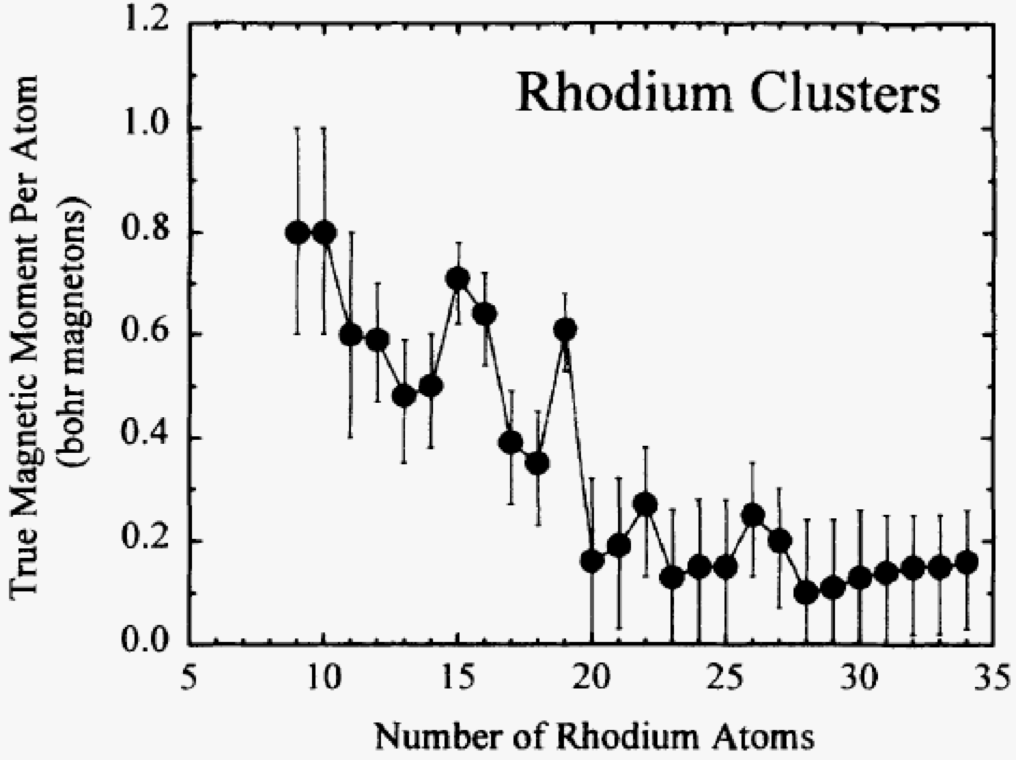


D. Qian et al., PRL87, 227204(2001)

See also V. Cros et al., Europhys. Lett. 49, 807 (2000)

Magnetic order versus dimensionality – Example: Fe

Rh turning magnetic in small clusters



A. J. Cox et al., Magnetism in 4d-transition metal clusters, Phys. Rev. B 49, 12295 (1994)

Physics

Narrowing of band

Stoner criterium

Quantum size effects

spintec SPIN IN ELECTRONICS

Magnetic anisotropy

Underlying physics

- Crystal electric field (CEF): Coulomb interaction between electronic orbitals and the crystal environment $\mathcal{H}_{\mathrm{CEF}}$
- Spin-orbit coupling S and L $\mathcal{H}_{\mathrm{SO}}$

	$\mathcal{H}_{\mathrm{CEF}}$	$\mathcal{H}_{\mathrm{SO}}$
3d	$1-10\ \mathrm{eV}$	$10-100\ \mathrm{meV}$
4f	$25\ \mathrm{meV}$	$100-500\ \mathrm{meV}$

Phenomenology

- Angular dependence of the energy of a magnetic material
- Applies to all orders: ferromagnets, antiferromagnets etc.
- Group theory predict terms in expansions:

Cubic $E_{\mathrm{mc}} = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^3 + \alpha_3^2\alpha_1^2) + K_2\alpha_1^2\alpha_2^2\alpha_3^2 + \cdots$

Hexagonal

$$E_{\mathrm{mc}} = K_1 \sin^2\theta + K_2 \sin^4\theta + K_3 \sin^6\theta + K_3' \sin^6\theta \sin^6\phi + \cdots$$

Numbers

- Low symmetry favors high anisotropy
- Large range of values in known materials

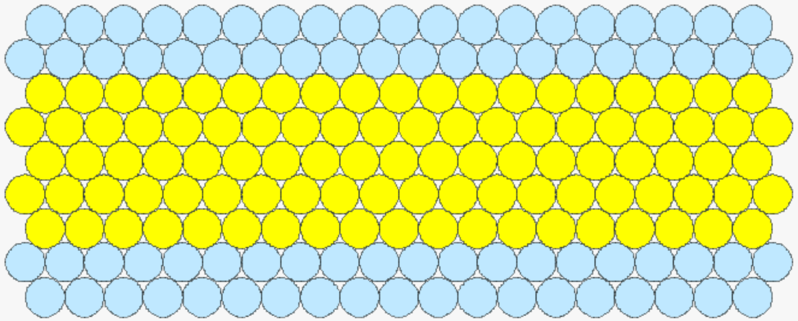
Crucial importance for applications

- Compass, spintronic-based magnetic sensors
- Magnetic recording, including tapes, hard-disk drives, magnetic random access memories

Lectures
Ester Palmero,
Yoichiro Tanaka

Interfacial magnetic anisotropy

Simple picture: interfacial magnetic anisotropy



- Breaking of symmetry for surface/interface atoms
- Brings a correction to magnetocrystalline anisotropy

$$E_s = K_{s,1}\cos^2\theta + K_{s,2}\cos^4\theta + \cdots$$

L. Néel, J. Phys. Radium 15, 15 (1954),
Superficial magnetic anisotropy and orientational superstructures

This surface energy, of the order of 0.1 to 1 erg/cm2, is liable to play a significant role in the properties of ferromagnetic materials spread in elements of dimensions smaller than 100Å.

Visionary !!!

First Experimental evidence

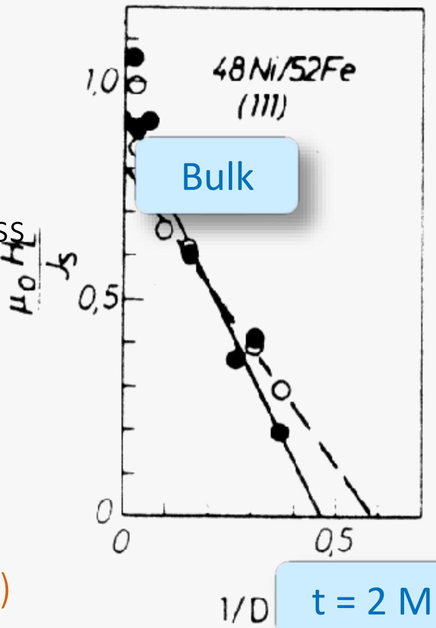
- Total anisotropy energy

$$\mathcal{E}(t) = K_V t + 2K_s$$

- Anisotropy per unit thickness

$$E(t) = K_V + \frac{2K_s}{t}$$

U. Gradmann and J. Müller,
Phys. Status Solidi 27, 313 (1968)



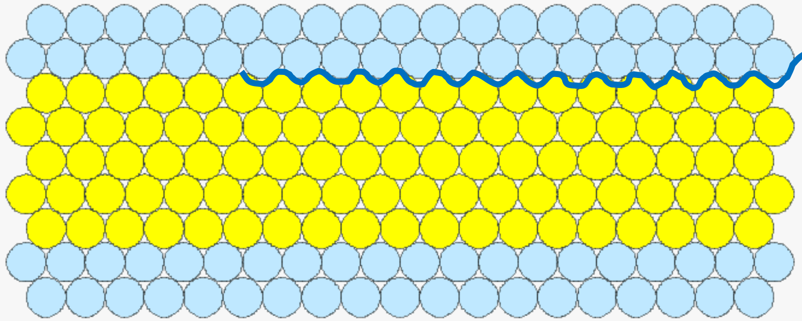
Figures

- Magnitude around $1\ \mathrm{mJ/m^2}$
- May promote perpendicular magnetization below thickness 1nm or so
- 80's & 90's: in contact with high spin-orbit materials (Au, Pt …)
- Since: Al_2O_3, MgO, graphene…

Lecture
Yoichiro Tanaka

Atomic contribution to magnetostatic energy

Surface contribution to shape anisotropy



Atomic-scale roughness

Interface dipolar anisotropy (per interface)

$$\mathcal{E}_{\mathrm{s,atom}} = -\mathrm{k_s}\delta\left(\frac{1}{2}\mu_0 M_s^2\right)\cos^2(\theta)$$

- Decreases slab demag coeff 1
- Change is large for open surface

Surface	k_s
fcc(111)	0.0344
fcc(001)	0.1178
hcc(110)	0.0383
bcc(001)	0.2187
hcp(0001)	0.0338

H.J.G. Draaisma et al., JAP 64, 3610 (1988)

Notes

- Included in Néel's pair interaction model from 1954 !
- Specific case of the dipolar crystalline anisotropy
- This effect is expected to be large in VdW materials, due to the large difference of in-plane versus out-of-plane distance between magnetic ions

P. Bruno, Physical origins and theoretical models of magnetic anisotropy, Ferienkurse des Forschungszentrums Jülich, Ch.24 (1993)

Lecture
Peng Li

Magneto-elastic anisotropy

Phenomenology

- Dependence of magnetic anisotropy on strain
- Can be viewed as the strain-derivative of magneto-crystalline anisotropy
- Source of
  - **Magnetostriction**: direction of magnetization induces strain
  - **Inverse magnetostriction**: strain tends to orient magnetization along specific directions
- Example: polycrystalline sample under uniaxial strain

$$E_{\mathrm{mel}} = -\lambda_{\mathrm{S}} \frac{E}{2}(3\cos^2\theta - 1)\epsilon - \frac{1}{2}E\epsilon^2 + \cdots$$

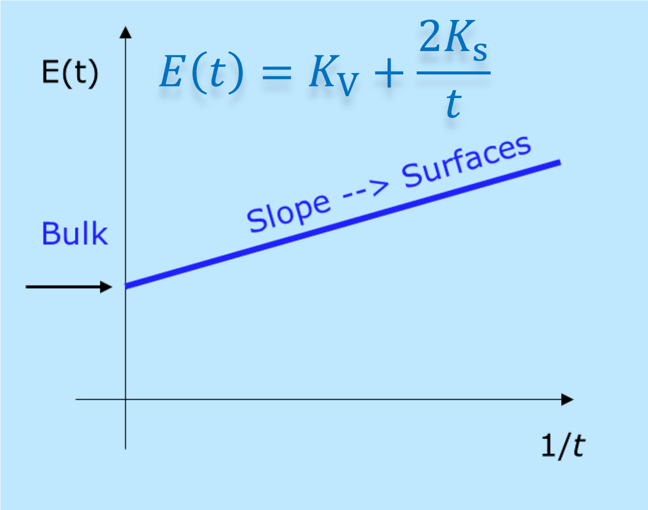
E Young modulus

Impact

- **Order of magnitude** of Lambda: 10^{-6}
- **Contributes to coercivity** in low-anisotropy materials
- Underpins effects such as **Invar**
- Magnetostriction is used in **actuators**

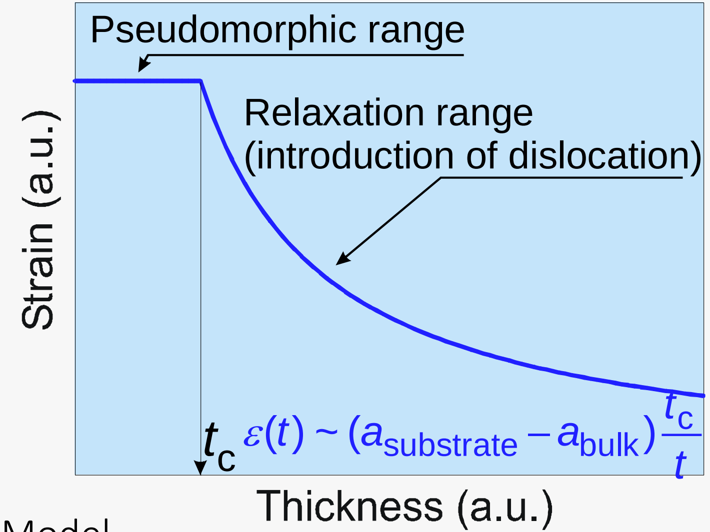
Magneto-elastic anisotropy – Thin films

Surface anisotropy alone



→ Bulk and surface anisotropy infered from intercept with *y* axis and slope

Inverse magnetostriction in films



Model

W. A. Jesser, Phys. Stat. Sol. 19, 95 (1967)

Experiments

U. Gradmann, Appl. Phys.3, 161 (1974)

→ Induces a 1/t variation of anisotropy, similar to interface contribution

C. Chappert and P. Bruno., JAP64, 5736 (1988)

Other effects…

- Interface roughness and intermixing
- Thickness-dependent thermal decay (when measured at T > 0K)
- Non-linearity of magneto-elastic coupling coefficients

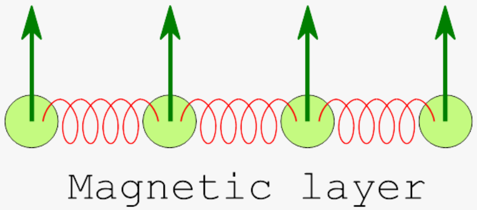
D. Sander, Rep. Prog. Phys. 62, 809 (1999)

U. Gradmann, Magnetism in ultrathin transition metal films, in Handbook of magnetic materials, vol. 7, Buschow, K. H. J. (Ed.), Elsevier (1993)

Chirality

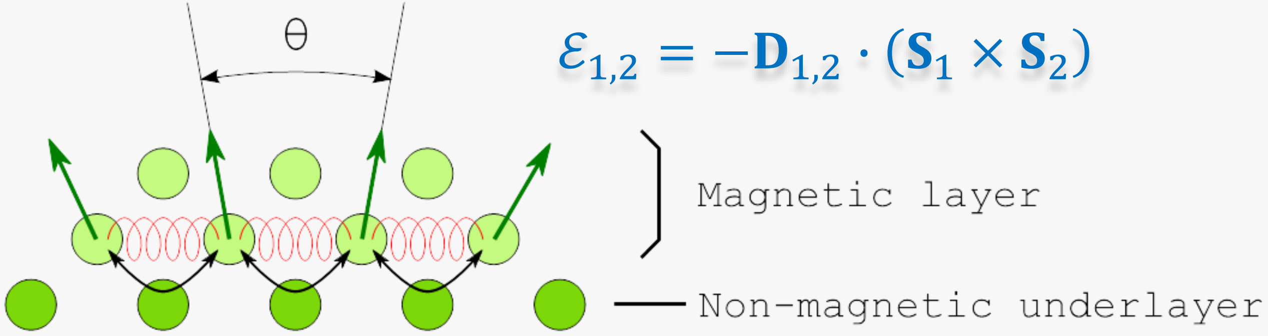
Magnetic exchange

$$\mathcal{E}_{1,2} = -J_{1,2}\,\mathbf{S}_1 \cdot \mathbf{S}_2$$

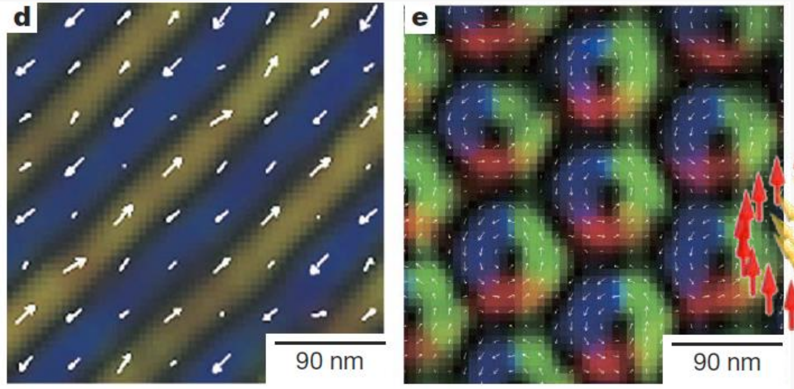


The Dzyaloshiinski-Moriha interaction (DMI)

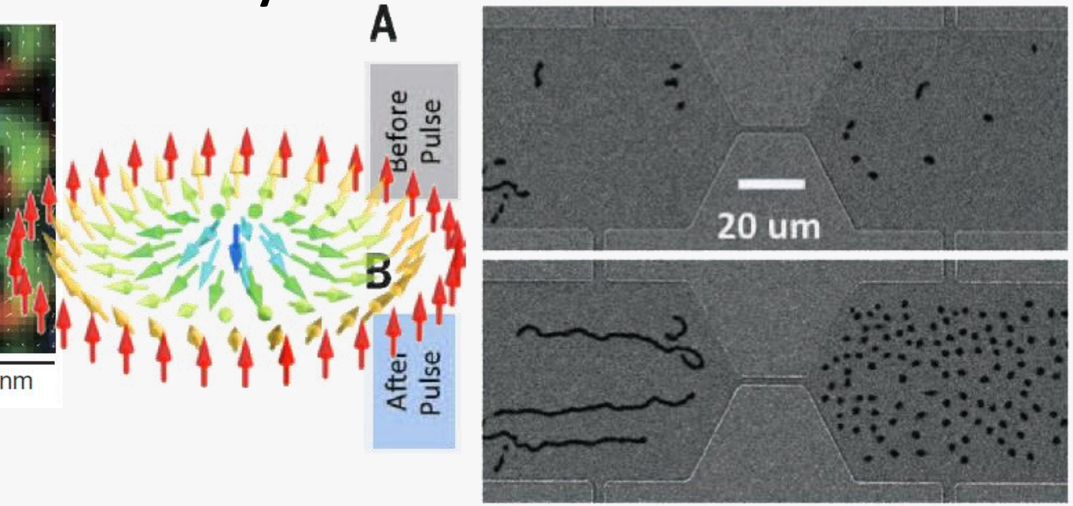
$$\mathcal{E}_{1,2} = -\mathbf{D}_{1,2} \cdot (\mathbf{S}_1 \times \mathbf{S}_2)$$



Chiral magnetization textures: spirals and skyrmions



X. Z. Yu et al., Nature 465, 901 (2010)



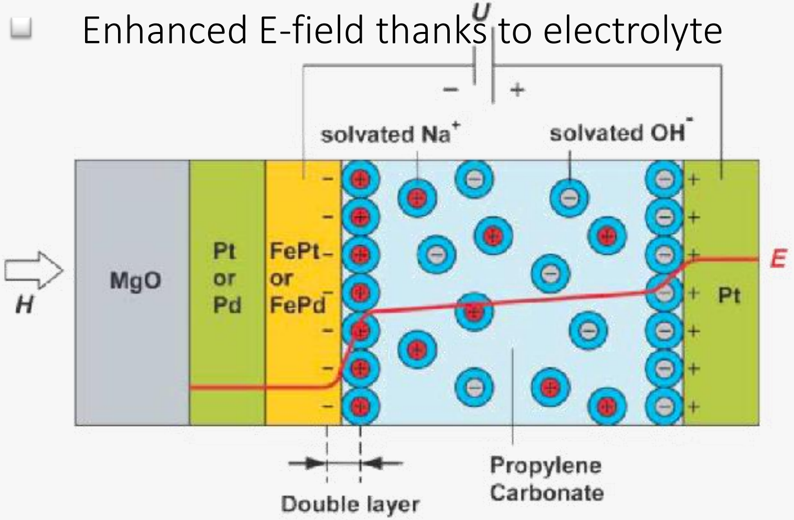
W. Jiang et al., Science 349, 283 (2015)

Voltage control of magnetization in metals

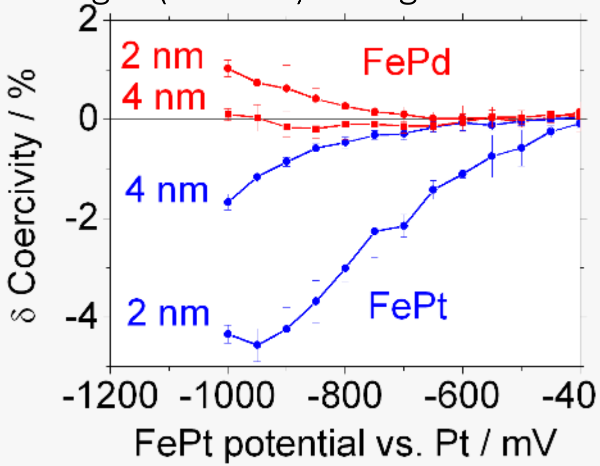
spintec SPIN IN ELECTRONICS

Seminal report

- Enhanced E-field thanks to electrolyte



- Slight (relative) change of coercivity



- Effect was not expected for metals, due to short screening length
- Relative change of coercivity is weak as coercivity is large

M. Weisheit et al., Science 315, 349 (2007)

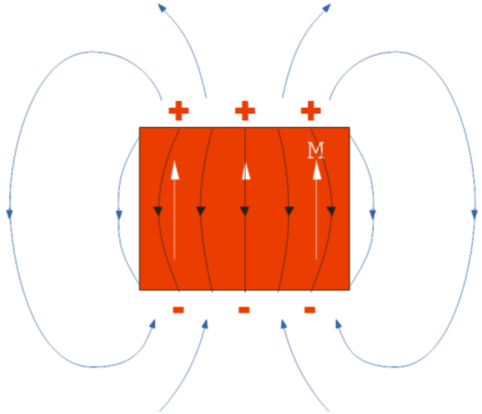
Developments

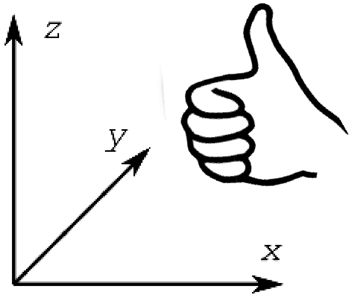
- Precessional switching with pulse of E-field
  Y. Shiota et al., Nature Mater.11, 39 (2012)
- Ferromagnetic resonance with ac E-field
  T. Nozaki et al., Nature Phys. 8, 491 (2012)
- Inversion of sign of DMI and skyrmions chirality
  R. Kumar et al., arXiv: 2009.13136 (2020)

Motivations for technology

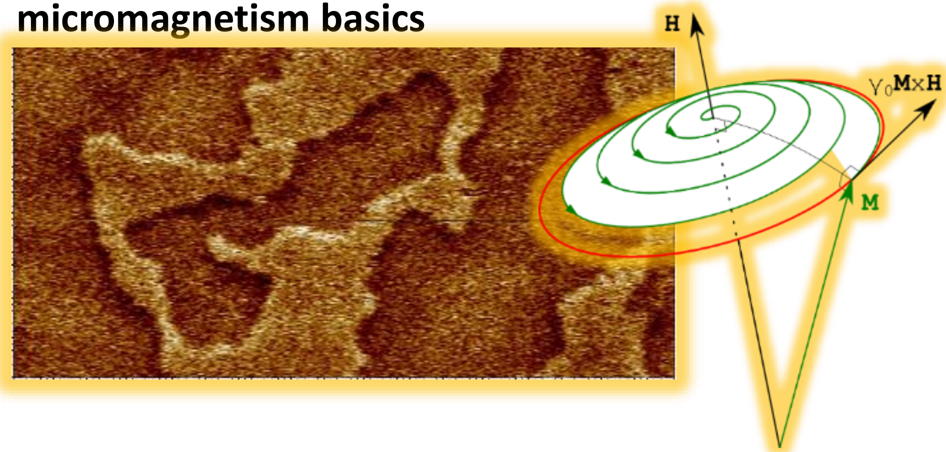
- Drastically reduce Joule heating (only capacitance current)
- Gateable functionality

- Units and fields

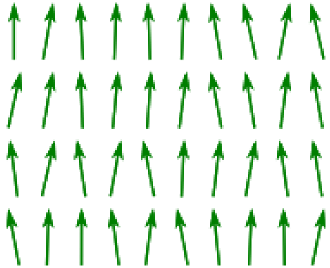




- Magnetization processes and micromagnetism basics



- Magnetic ordering and materials

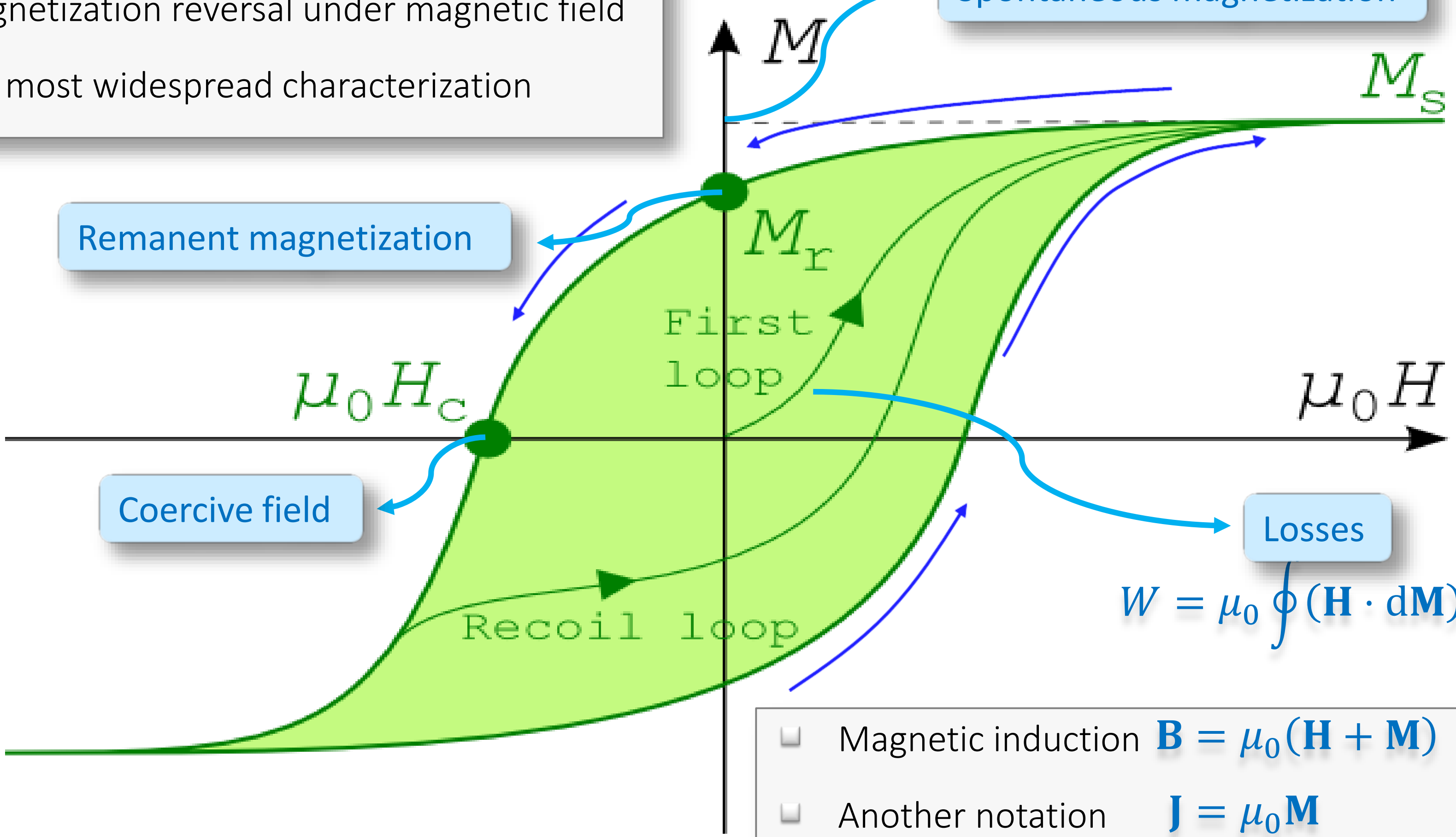


The hysteresis loop

The hysteresis loop

- Magnetization reversal under magnetic field
- The most widespread characterization

Spontaneous ≠ Saturation



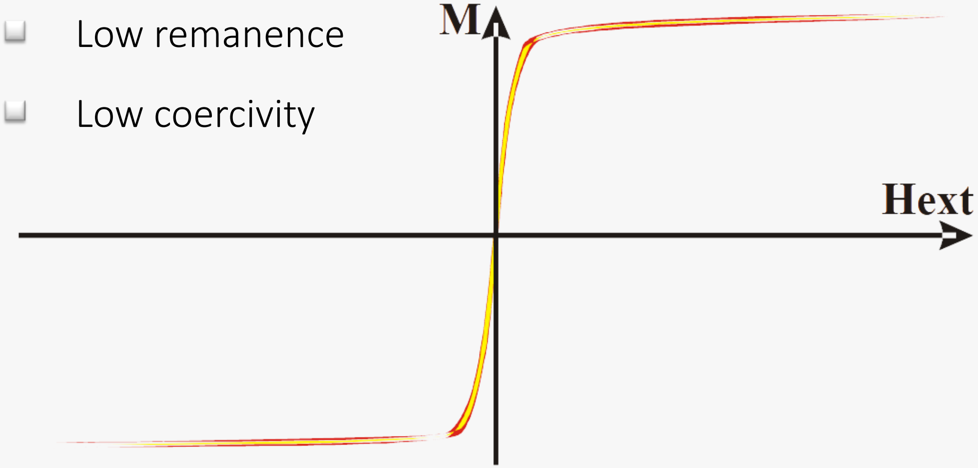
$$W = \mu_0 \oint (\mathbf{H} \cdot \mathrm{d}\mathbf{M})$$

- Magnetic induction $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$
- Another notation $\mathbf{J} = \mu_0 \mathbf{M}$

The hysteresis loop

Soft-magnetic materials

- Low remanence
- Low coercivity

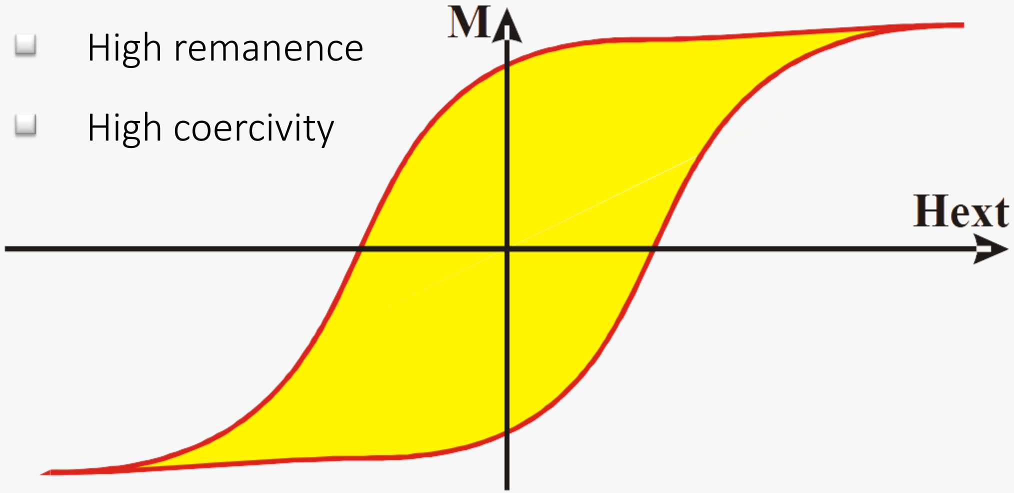


Applications

- Transformers
- Flux guides, sensors
- Magnetic shielding

Hard-magnetic materials

- High remanence
- High coercivity



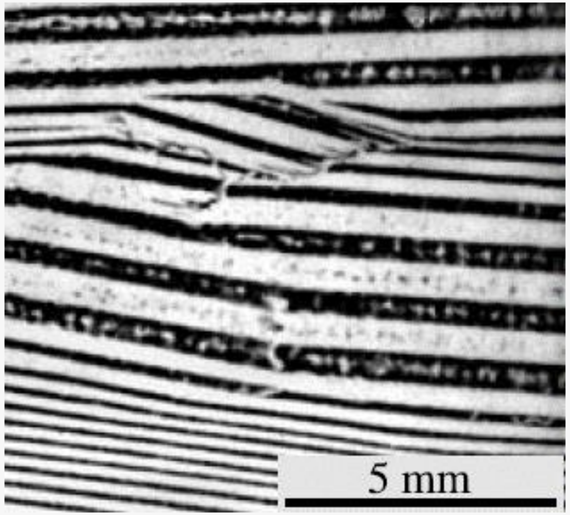
Applications

- Permanent magnets,
- Motors and generators
- Magnetic recording

The origin of magnetic domains

Historical background

- **Puzzle from the early days** of magnetism: some materials may be magnetized under applied field, however "loose" their magnetization when the field is removed
- **Postulate from Weiss**: existence of magnetic domains, i.e., large (3D) regions with each uniform magnetization
- **Magnetic domain walls** are the narrow (2D: planes) regions separating neighboring domains

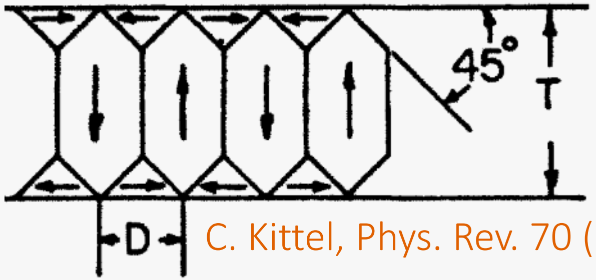


FeSi sheet (transformer)

A. Hubert, magnetic domains

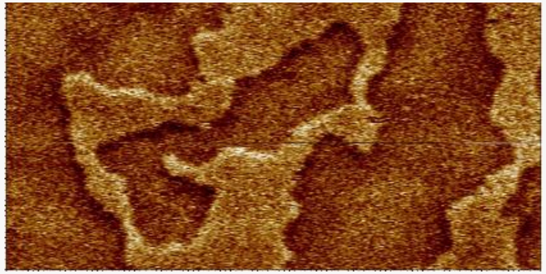
Origin of domains

- **Minimization of energy:** closure of magnetic flux to decrease dipolar energy, at the expense of energy in the domain walls (exchange, anisotropy…)



C. Kittel, Phys. Rev. 70 (11&12), 965 (1946)

- **Magnetic history:** magnetic domains along various directions may form through the ordering transition or following a partial magnetization process, persisting even though leaving the system not in the ground state



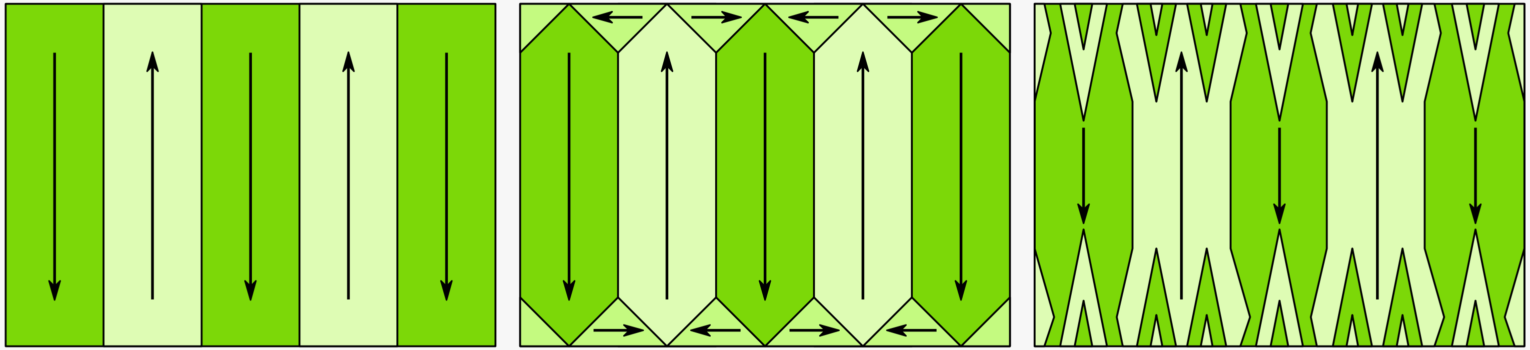
MgO\Co[1nm]\Pt

Magnetic Force Microscopy, 5 x 2.5 µm

Statics – Tendency for flux-closure domains

Films with easy axis out-of-the-plane: Kittel domains

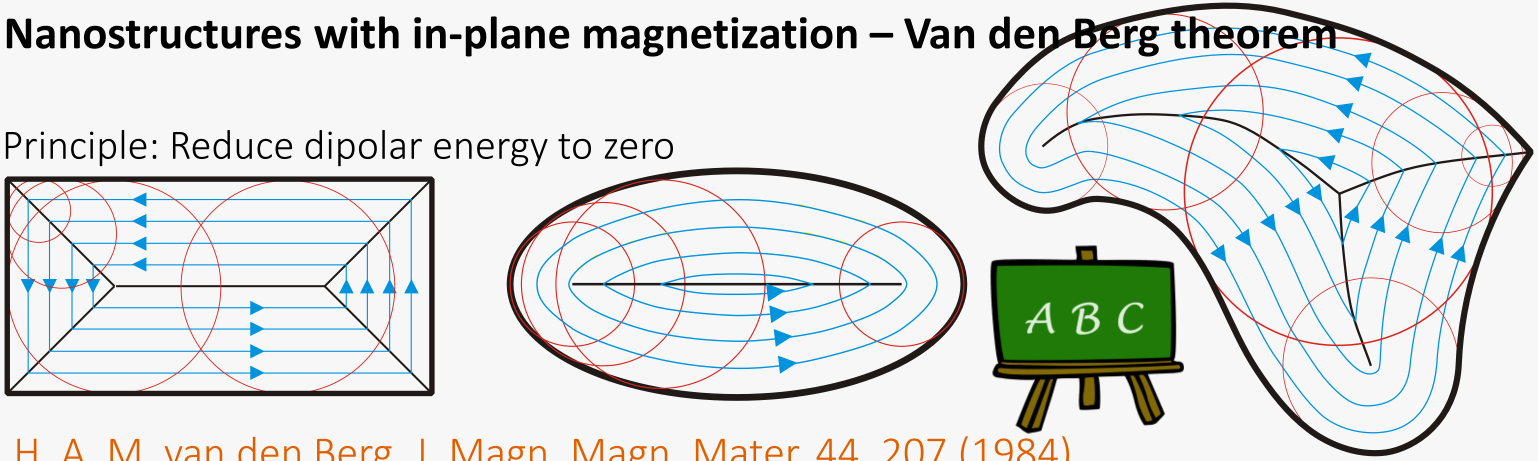
Principle: compromise between gain in dipolar energy, and cost in wall energy



C. Kittel, Physical theory of ferromagnetic domains, Rev. Mod. Phys. 21, 541 (1949)

Nanostructures with in-plane magnetization – Van den Berg theorem

Principle: Reduce dipolar energy to zero

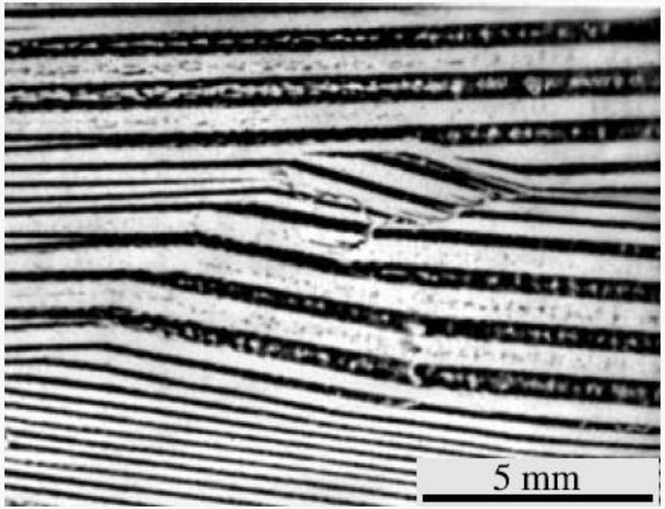


H. A. M. van den Berg, J. Magn. Magn. Mater. 44, 207 (1984)

Magnetic domains from bulk to nano

Bulk materials

Numerous and complex magnetic domains

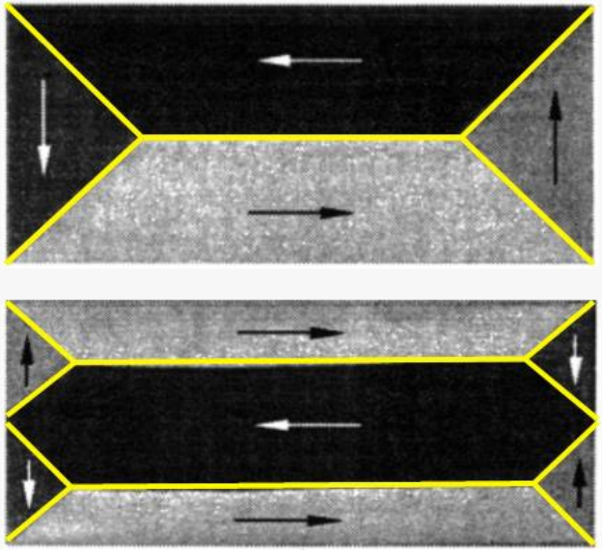


FeSi sheet (transformer)

A. Hubert, magnetic domains

Mesoscopic scale

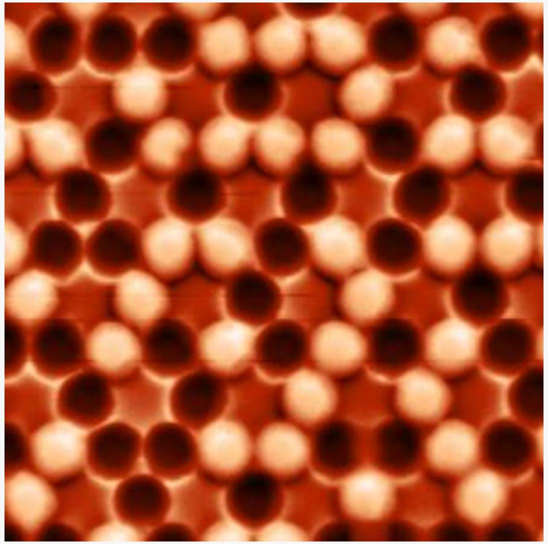
Small number of domains, simple shape



Microfabricated dots,
Kerr magnetic imaging
A. Hubert, magnetic domains

Nanometric scale

Magnetic single domain



Microfabricated dots,
magnetic force microscopy
Sample courtesy: I. Chioar

The Stoner-Wohlfarth model

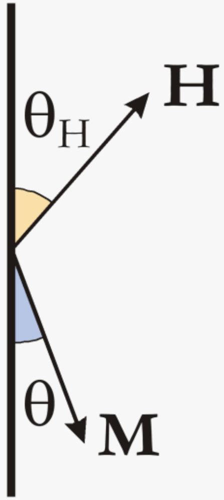
Framework: uniform magnetization

- Drastic, unsuitable in most cases
- Remember: demagnetization field may not be uniform

$$\mathcal{E} = E\mathcal{V}$$

$$= \mathcal{V}[K_{\mathrm{eff}} \sin^2\theta - \mu_0 M_s H \cos(\theta - \theta_H)]$$

- Anisotropy: $K_{\mathrm{eff}} = K_{\mathrm{mc}} + (\Delta N)K_{\mathrm{d}}$



Names used

- Uniform rotation / magnetization reversal
- Coherent rotation / magnetization reversal
- Macrospin etc.

Dimensionless units

$$e = \sin^2\theta - 2h\cos(\theta - \theta_H)$$

$$e = \mathcal{E}/(K\mathcal{V})$$

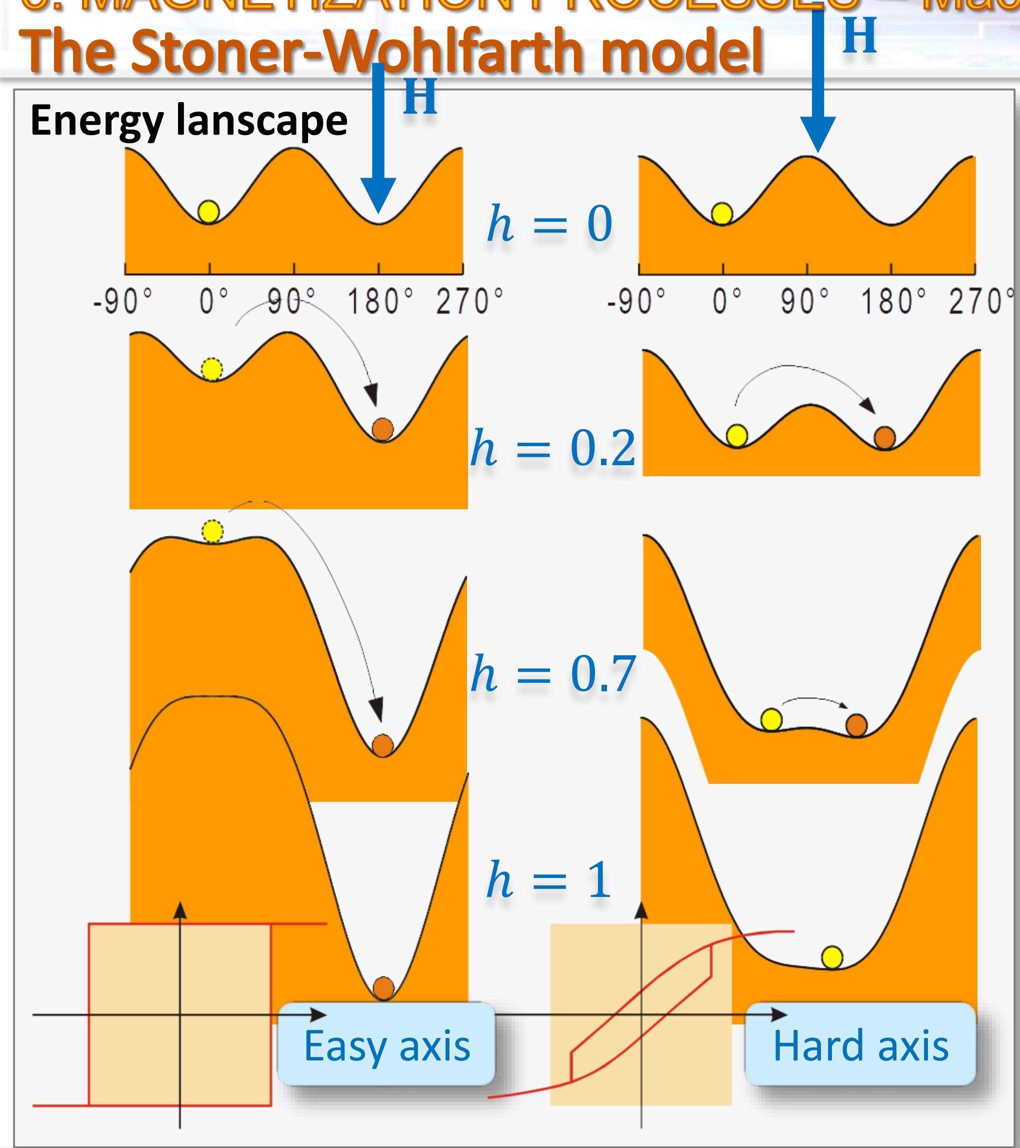
$$h = H/H_{\mathrm{a}}$$

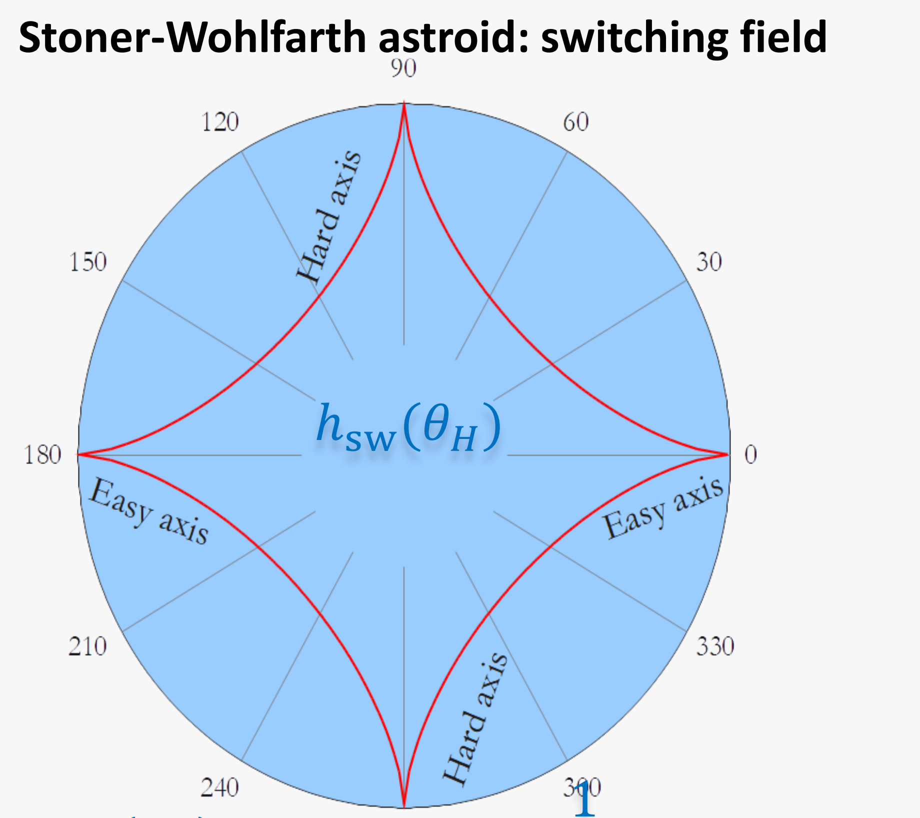
$$H_{\mathrm{a}} = 2K/(\mu_0 M_s)$$

L. Néel, Compte rendu Acad. Sciences 224, 1550 (1947)

E. C. Stoner and E. P. Wohlfarth,
Phil. Trans. Royal. Soc. London A240, 599 (1948)
Reprint: IEEE Trans. Magn. 27(4), 3469 (1991)

The Stoner-Wohlfarth model





$$h_{\mathrm{sw}}(\theta_H) = \frac{1}{\left(\sin^{2/3}\theta_H + \cos^{2/3}\theta_H\right)^{3/2}}$$

J. C. Slonczewski, Research Memo RM 003.111.224, IBM Research Center (1956)

Switching field versus coercive field

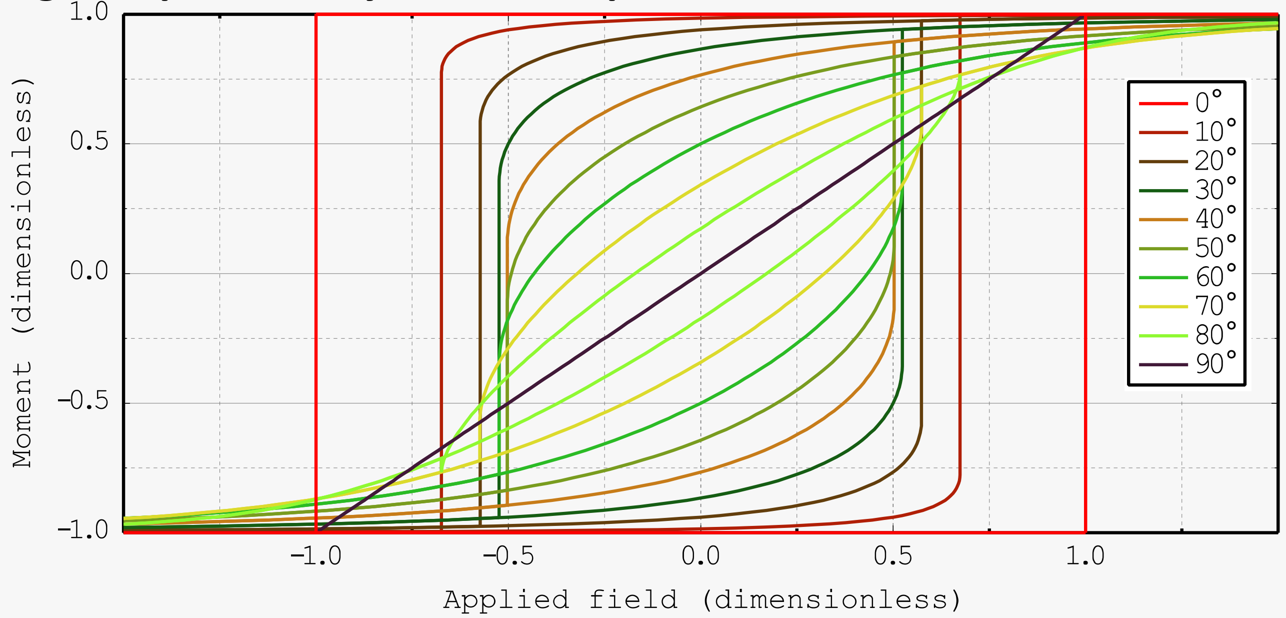
Switching field H_{sw}

- A value of field at which an irreversible (abrupt) jump of magnetization angle occurs.
- Can be measured only in single particles.

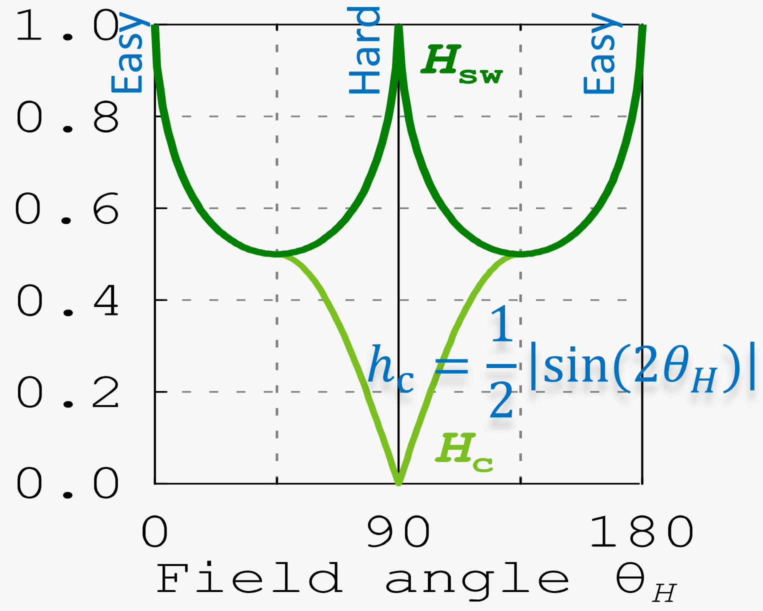
Coercive field H_c

- The field at which $\mathbf{H}\cdot\mathbf{M}=0$
- Measurable in materials (large number of 'particles').
- May or may not be a measure of the mean switching field at the microscopic level

Angle-dependent hysteresis loops

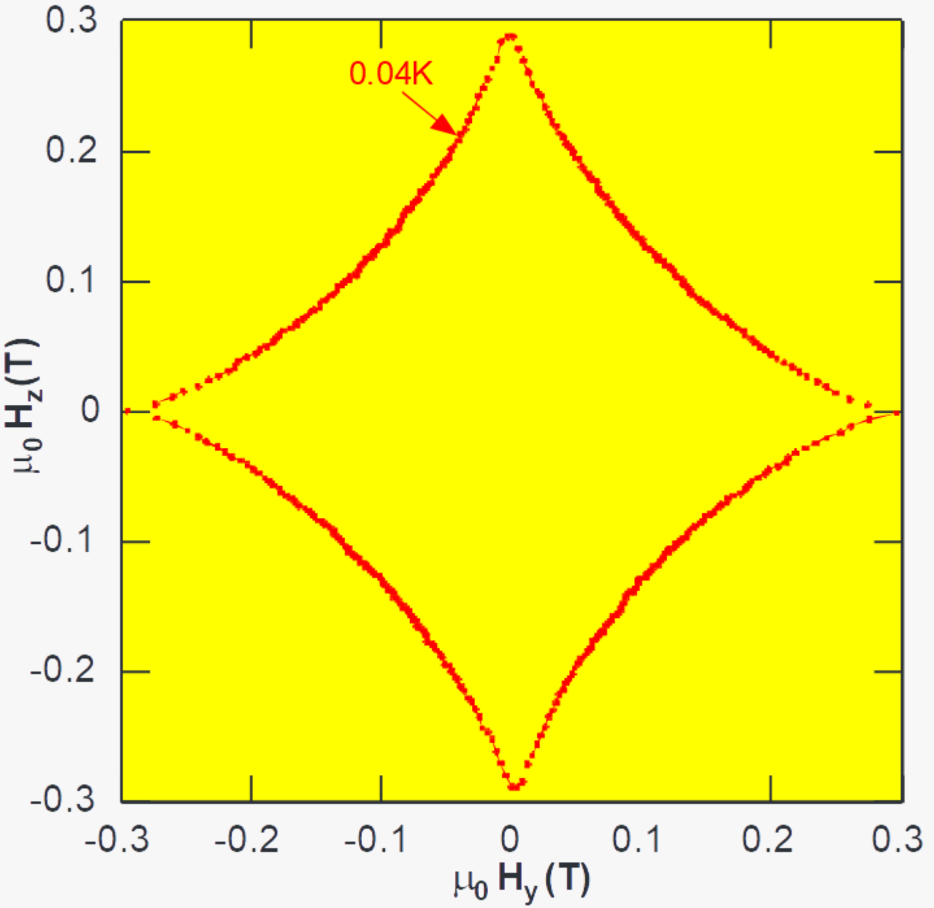


Switching versus coercive field

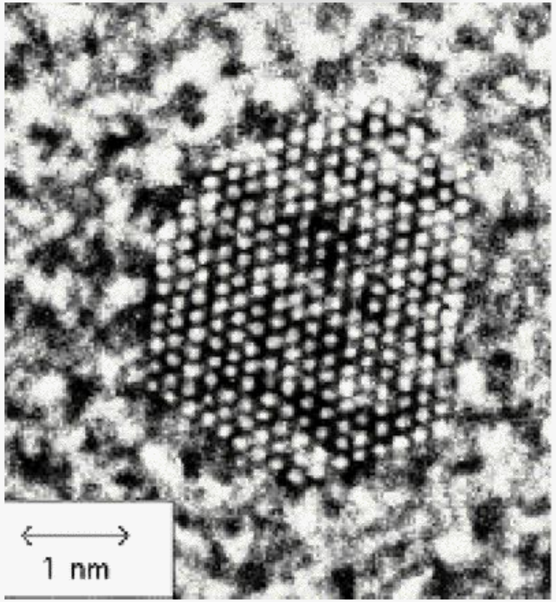


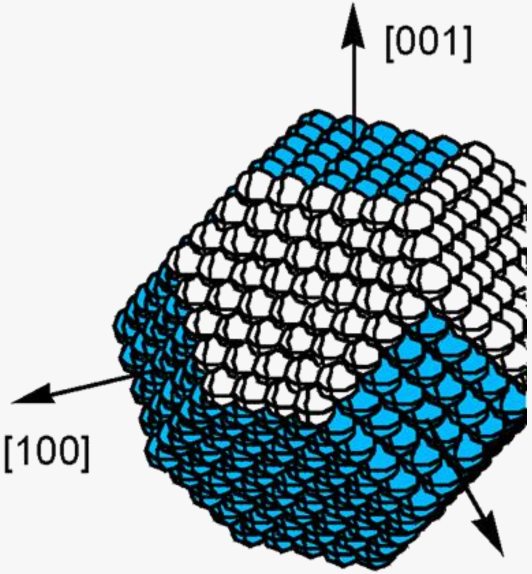
Experiments

First experimental evidence



Co cluster

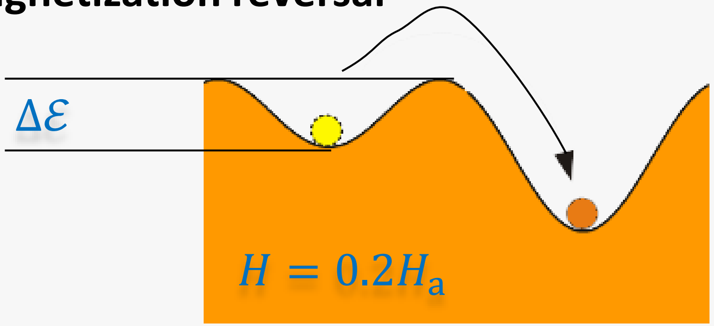




W. Wernsdorfer et al., Phys. Rev. Lett. 78, 1791 (1997)

Superparamagnetism and the blocking temperature

Energy barrier preventing magnetization reversal



$$\Delta\mathcal{E} = K\mathcal{V}\left(1 - \frac{H}{H_a}\right)^2$$

E. F. Kneller, J. Wijn (ed.) Handbuch der Physik XIII/2: Ferromagnetismus, Springer, 438 (1966)

M. P. Sharrock, J. Appl. Phys. 76, 6413-6418 (1994)

Thermal activation

Brown, Phys.Rev.130, 1677 (1963)

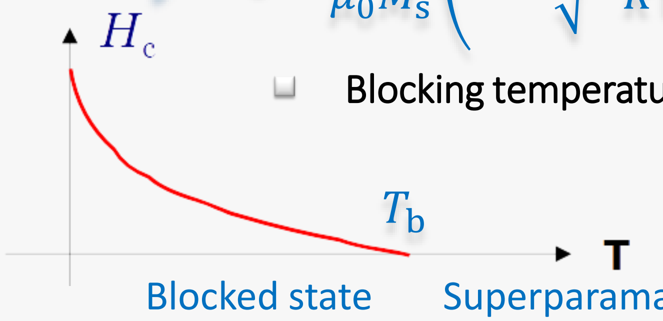
- **Waiting time** (Arrhenius law) $\tau = \tau_0 \exp\left(\frac{\Delta\mathcal{E}}{k_B T}\right)$

  ➡ $\Delta\mathcal{E} = k_B T \ln\left(\frac{\tau}{\tau_0}\right)$

- Lab measurement: $\tau \approx 1\,\mathrm{s}$ ➡ $\Delta\mathcal{E} \approx 25 k_B T$

➡ $H_c = \frac{2K}{\mu_0 M_s}\left(1 - \sqrt{\frac{25 k_B T}{K\mathcal{V}}}\right)$

- Blocking temperature: $T_b \approx \frac{K\mathcal{V}}{25 k_B}$



Lecture Sara Majetich

- Coercivity and remanence are lost at small size
- Incentive to enhance magnetic anisotropy

The case of magnetic recording or memory

$\tau \approx 10^9\,\mathrm{s}$ ➡ $K\mathcal{V}_b \approx 40-60\,k_B T$

Superparamagnetism – Modeling

Formalism

- Energy

$$\mathcal{E} = K\mathcal{V}\mathrm{f}(\theta,\phi) - \mu_0 \mu H$$

- Partition function

$$Z = \sum \exp(-\beta\mathcal{E})$$

- Average moment

$$\langle\mu\rangle = \frac{1}{\beta\mu_0 Z}\frac{\partial Z}{\partial H}$$

Isotropic case

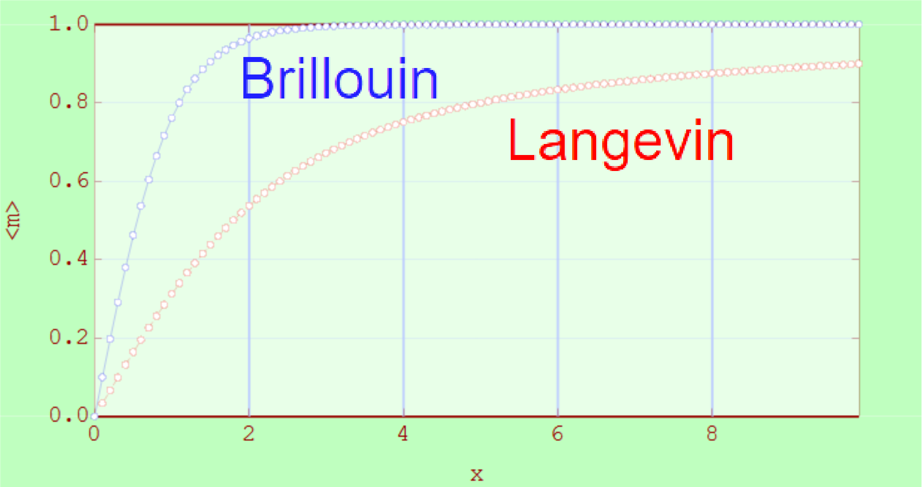
$$Z = \int_{-\mathcal{M}}^{\mathcal{M}} \exp(-\beta\mathcal{E})\,\mathrm{d}\mu$$

Note: equivalent to integrate on solid angle

$$\langle\mu\rangle = \mathcal{M}\left[\coth(x) - \frac{1}{x}\right]$$

Langevin function

Note: refers to the moment of the particle, not a spin ½



Highly anisotropic case

$$Z = \exp(\beta\mu_0\mathcal{M}H) + \exp(-\beta\mu_0\mathcal{M}H)$$

Note: only two states are populated, 'up' and 'down'

$$\langle\mu\rangle = \mathcal{M}\tanh(x)$$

Brillouin ½ function

- Fit M(H) curve to extract magnetization (and hence the volume) of nanoparticles
- Beware of anisotropy strength and distribution in fits !

Formalism

Magnetization

Magnetization vector **M**

- Continuous function
- May vary over time and space
- Modulus is constant and uniform (hypothesis in micromagnetism)

$$\mathbf{M}(\mathbf{r}) = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_s \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$$

$$m_x^2 + m_y^2 + m_z^2 = 1$$

Mean field approach is possible: $M_s = M_s(T)$

Lecture
Denys Makarov

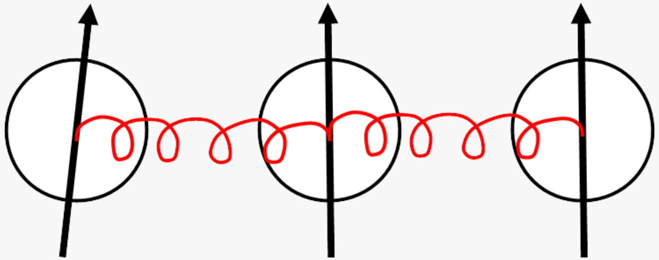
Exchange interaction

- Atomistic view $\mathcal{E} = -\sum_{i \neq j} J_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j$ (total energy, J)
- Micromagnetic view $\mathbf{S}_i \cdot \mathbf{S}_j = S^2 \cos(\theta_{i,j}) \approx S^2 \left(1 - \frac{\theta_{i,j}^2}{2}\right)$

$$E_{\mathrm{ex}} = A(\nabla \cdot \mathrm{m})^2 = A \sum_{i,j} \left(\frac{\partial m_i}{\partial x_j}\right)^2$$

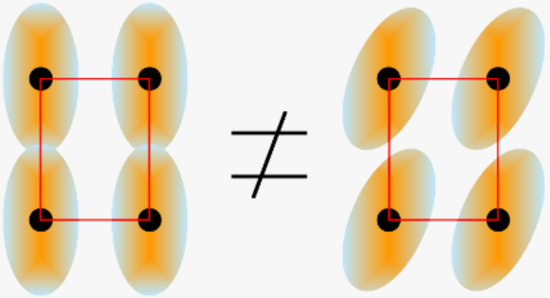
The various types of magnetic energy

Exchange energy



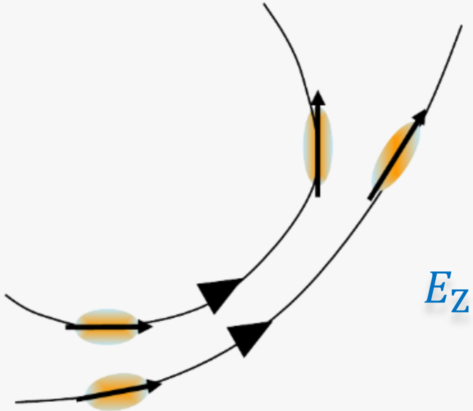
$$E_{\mathrm{ex}} = A(\boldsymbol{\nabla}\cdot\mathrm{m})^2 = A\sum_{i,j}\left(\frac{\partial m_i}{\partial x_j}\right)^2$$

Magnetocrystalline anisotropy energy



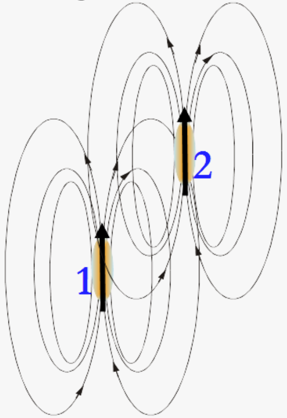
$$E_{\mathrm{mc}} = K\,f(\theta,\varphi)$$

Zeeman energy (→ enthalpy)



$$E_{\mathrm{Z}} = -\mu_0\mathbf{M}.\mathbf{H}$$

Magnetostatic energy



$$E_{\mathrm{d}} = -\frac{1}{2}\mu_0\mathbf{M}\cdot\mathbf{H}_{\mathrm{d}}$$

Magnetic length scales (analytics)

The dipolar exchange length

When: anisotropy and exchange compete

$$E = \underbrace{A\left(\frac{\partial m_i}{\partial x_j}\right)^2}_{\text{Exchange}} + \underbrace{K_\mathrm{d}\sin^2\theta}_{\text{Dipolar}}$$

$\mathrm{J/m}$ $\quad$ $\mathrm{J/m^3}$ $\quad$ $K_\mathrm{d} = \frac{1}{2}\mu_0 M_\mathrm{s}^2$

$$\boxed{\Delta_\mathrm{d} = \sqrt{A/K_\mathrm{d}} = \sqrt{2A/\mu_0 M_\mathrm{s}^2}}$$

$\Delta_\mathrm{d} \simeq 3 - 10\ \mathrm{nm}$

Critical single-domain size, relevant for small particles made of soft magnetic materials

Often called: exchange length

The anisotropy exchange length

When: anisotropy and exchange compete

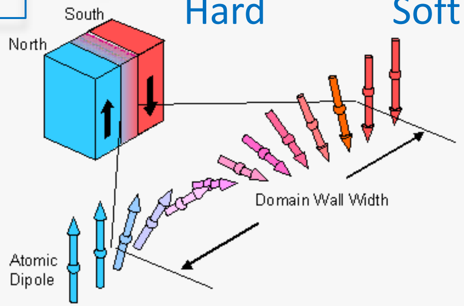
$$E = \underbrace{A\left(\frac{\partial m_i}{\partial x_j}\right)^2}_{\text{Exchange}} + \underbrace{K\sin^2\theta}_{\text{Anisotropy}}$$

$\mathrm{J/m}$ $\quad$ $\mathrm{J/m^3}$

$$\boxed{\Delta_\mathrm{u} = \sqrt{A/K}}$$

$\Delta_\mathrm{u} \simeq 1\ \mathrm{nm} \rightarrow 100\ \mathrm{nm}$

Hard $\quad$ Soft



Sometimes called: Bloch parameter, or wall width

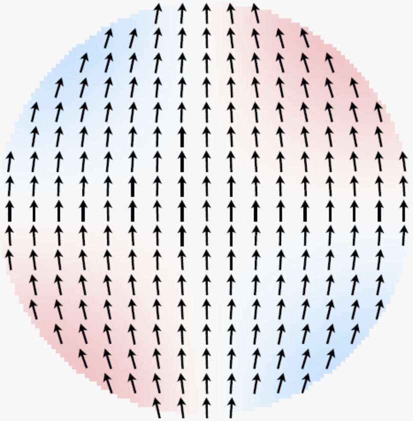
Note: Other length scales can be defined, e.g. with magnetic field

Micromagnetic simulation

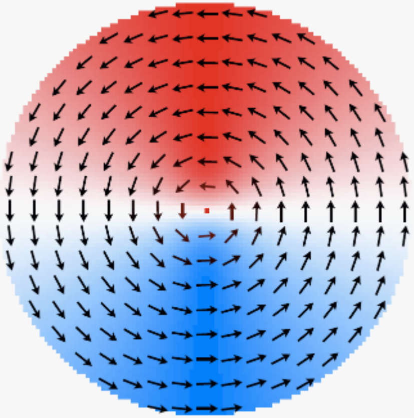
Principle

- Subdivides a system in small prisms or tetrahedrons
- Considers all energies
- Solves the Landau-Lifshitz equation

Flat disk

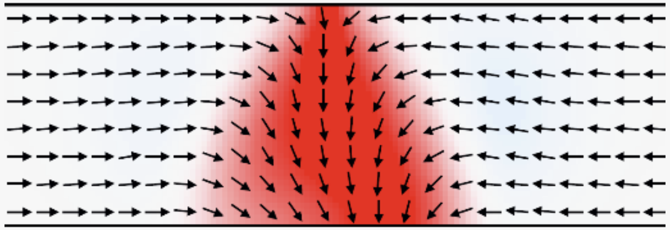


Near single-domain



Vortex state

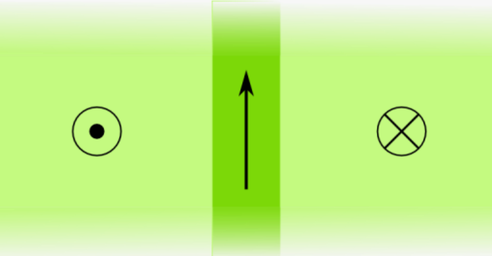
Domain wall in a flat strip



Transverse domain wall

Magnetic domains walls (and dimensionality)

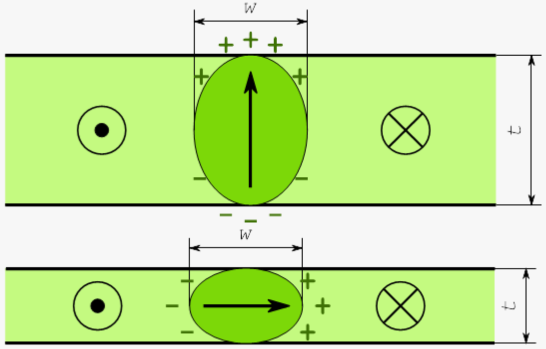
Bloch wall in the bulk (2D)



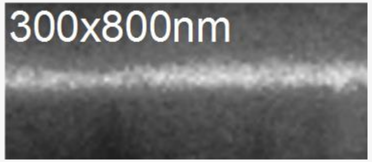
- No magnetostatic energy
- Width $\Delta_{\mathrm{u}} = \sqrt{A/K}$
- Energy $\gamma_{\mathrm{W}} = 4\sqrt{AK}$

F. Bloch, Z. Phys. 74, 295 (1932)

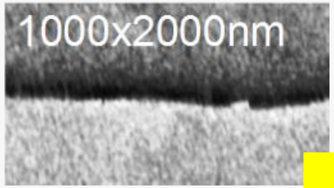
Domain walls in thin films (towards 1D)



Bloch wall $t \gtrsim w$



Néel wall $t \lesssim w$



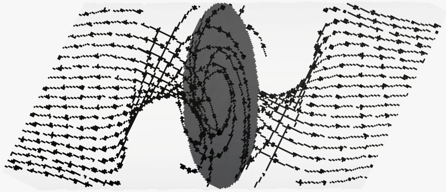
- Implies magnetostatic energy
- No exact analytic solution

L. Néel, C. R. Acad. Sciences 241, 533 (1956)

Lecture Manuel Vazquez

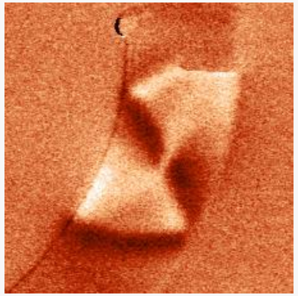
Bloch point (0D)

- Point with vanishing magnetization



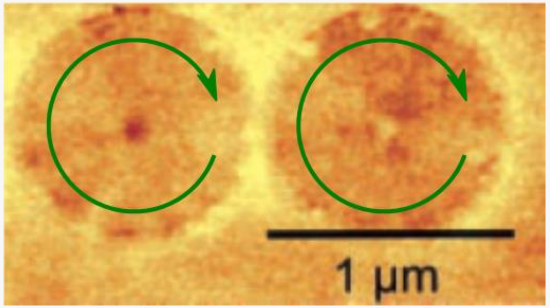
W. Döring, JAP 39, 1006 (1968)

Constrained walls (eg in strips)



Permalloy (15nm)
Strip width 500nm

Vortex (1D → 0D)

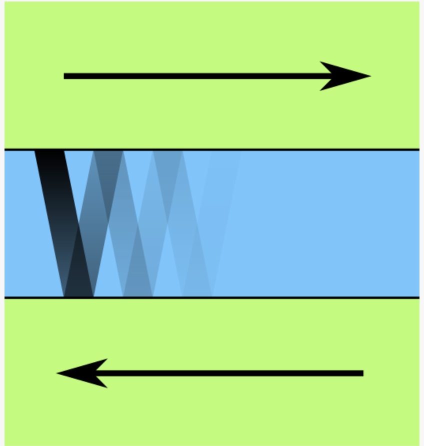


T. Shinjo et al., Science 289, 930 (2000)

Interlayer exchange coupling

spintec
SPIN IN ELECTRONICS

Spin-dependent quantum confinement



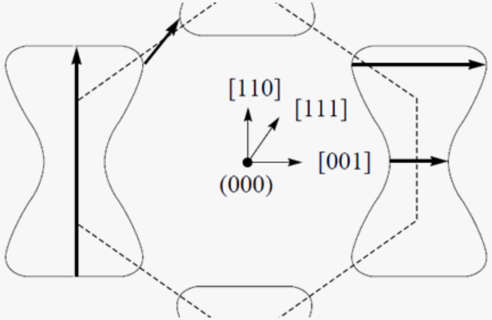
- Forth & back phase shift

$$\Delta\varphi = qt + \varphi_A + \varphi_B$$

- Spin dependence:

$$r_A, \varphi_A, r_B, \varphi_B$$

Oscillating constructive and destructive interferences with spacer thickness



Cu Fermi surface

- Importance of nesting
- Depends on crystal direction

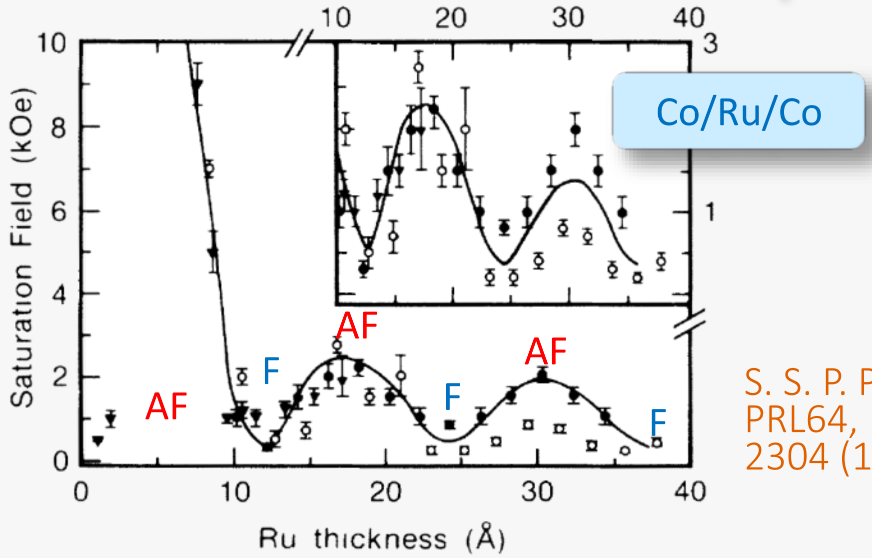
P. Bruno, J. Phys. Condens. Matter 11, 9403 (1999)

Coupling strength

$E_s(t) = J(t)\cos\theta$ with unit: $\mathrm{J/m^2}$

$$\theta = \langle \mathbf{m}_1, \mathbf{m}_2 \rangle$$

$$J(t) = \frac{A}{t^2}\sin(q_\alpha t + \Psi)$$

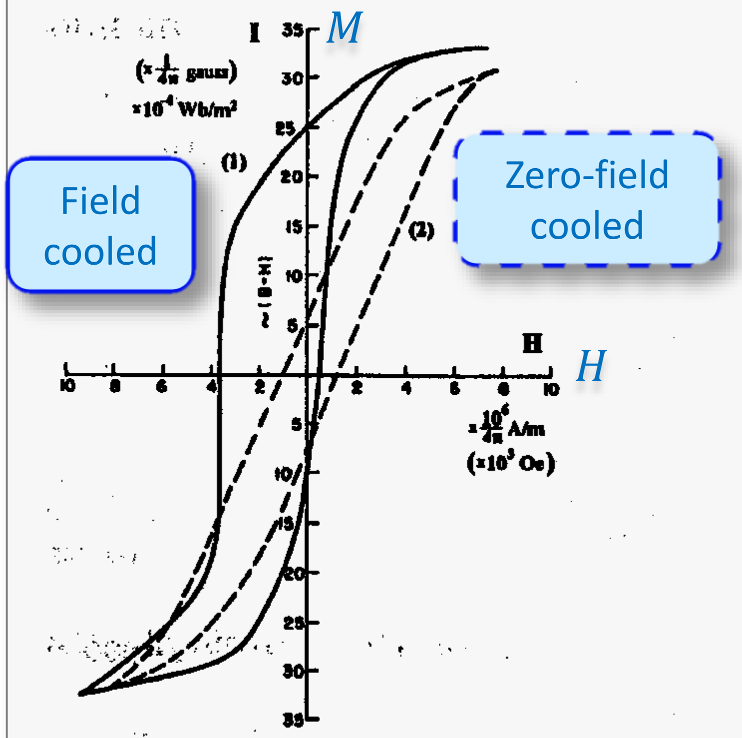


S. S. P. Parkin et al., PRL64, 2304 (1990)

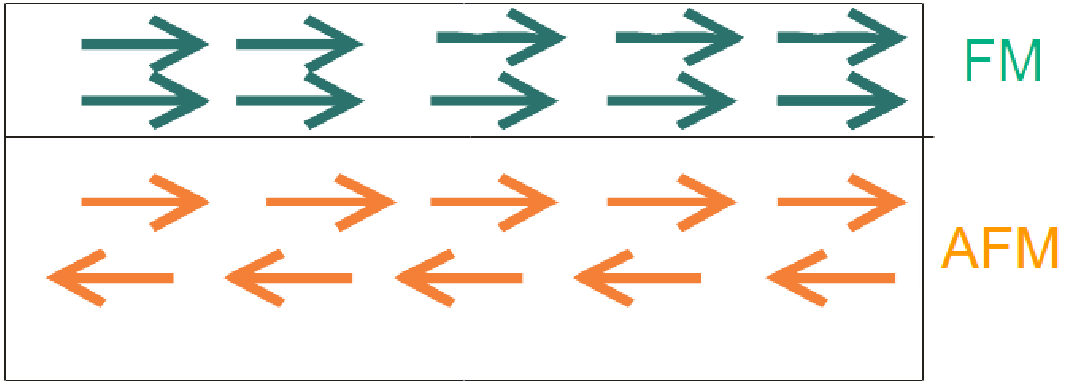
- RKKY = Ruderman-Kittel-Kasuya-Yoshida
- A function quantum effect at room temperature !
- Crucial to couple magnetic layers in stacks

Exchange bias

Seminal investigation



Meiklejohn and Bean, Phys. Rev. 102, 1413 (1956), Phys. Rev. 105, 904, (1957)



- Field-shift of hysteresis loop
- Increase of coercivity
- Crucial to design reference layer in memories

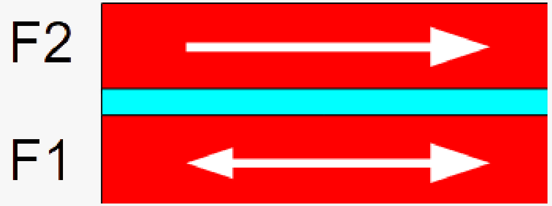
Exchange bias, J. Nogués and Ivan K. Schuller, J. Magn. Magn. Mater. 192 (1999) 203

Exchange anisotropy—a review, A E Berkowitz and K Takano, J. Magn. Magn. Mater. 200 (1999)

Synthetic antiferromagnets and spin valves

spintec
SPIN IN ELECTRONICS

RKKY Synthetic Ferrimagnets (SyF) – Basics



- Crude phenomenology

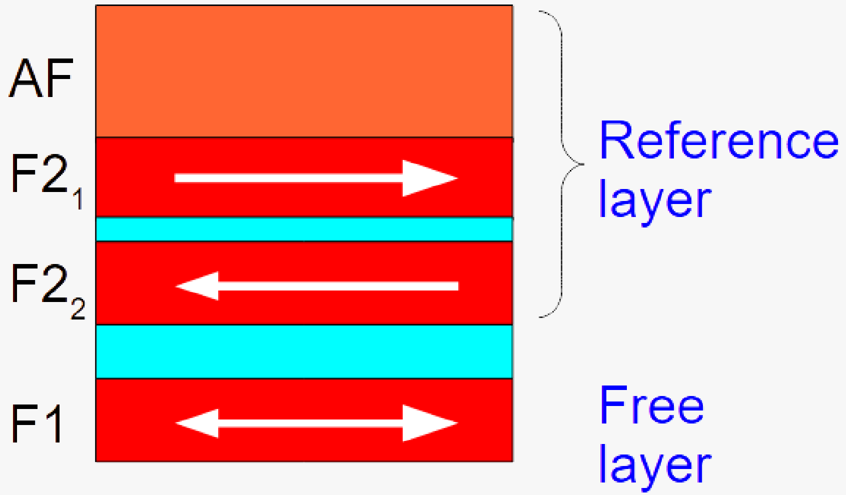
$$M = \frac{|e_1 M_1 - e_2 M_2|}{e_1 + e_2} \qquad K = \frac{e_1 K_1 + e_2 K_2}{e_1 + e_2}$$

$$\Rightarrow H_c \approx \frac{e_1 M_1 H_{c,1} + e_2 M_2 H_{c,2}}{|e_1 M_1 - e_2 M_2|}$$

- Enhances coercivity
- Reduces cross-talk in dense arrays

Spin valves

- “Free” and reference layers



B. Diény et al., J. Magn. Magn. Mater. 93, 101 (1991)

Lectures
Susana Cardoso Freitas
Yoichiro Tanaka

- Spin-valves are key elements in magneto-resistive devices (sensors, memories)
- Control Ru thickness within the Angström !

Magnetization switching of extended systems

Brown paradox

In most (extended systems): $H_c \ll \dfrac{2K}{\mu_0 M_s}$

(Micromagnetic) modeling

Exhibit analytic, nevertheless realistic models for magnetization reversal

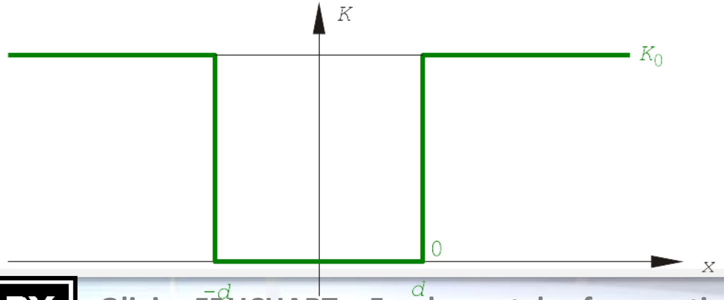
PHYSICAL REVIEW VOLUME 119, NUMBER 1 JULY 1, 1960

Reduction in Coercive Force Caused by a Certain Type of Imperfection

A. AHARONI
Department of Electronics, The Weizmann Institute of Science, Rehovot, Israel

(Received February 1, 1960)

As a first approach to the study of the dependence of the coercive force on imperfections in materials which have high magnetocrystalline anisotropy, the following one-dimensional model is treated. A material which is infinite in all directions has an infinite slab of finite width in which the anisotropy is 0. The coercive force is calculated as a function of the slab width. It is found that for relatively small widths there is a considerable reduction in the coercive force with respect to perfect material, but reduction saturates rapidly so that it is never by more than a factor of 4.



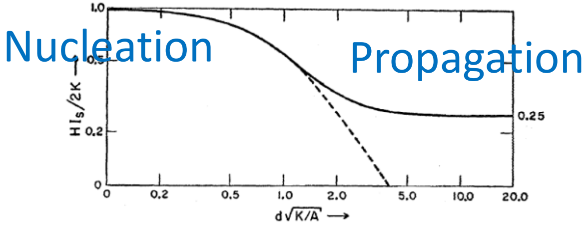
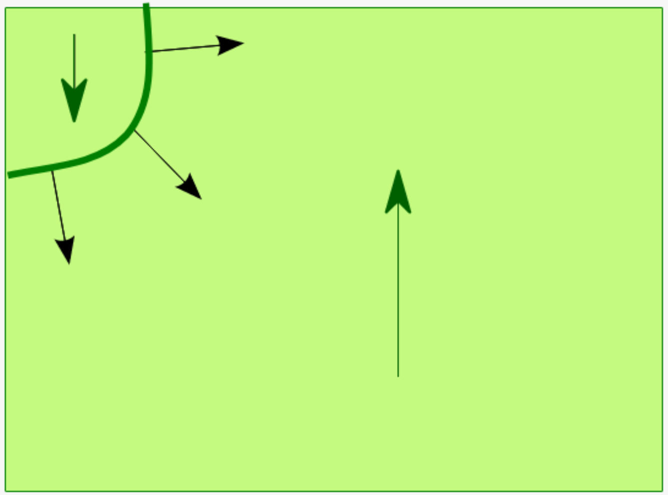


FIG. 1. The nucleation field (dashed) and coercive force (full curve) in terms of the coercive force of perfect material, $HI_s/2K$, as functions of the defect size, d.

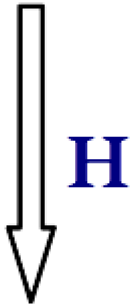
Nucleation – Propagation mechanisms

How are domain walls involved in magnetization reversal?

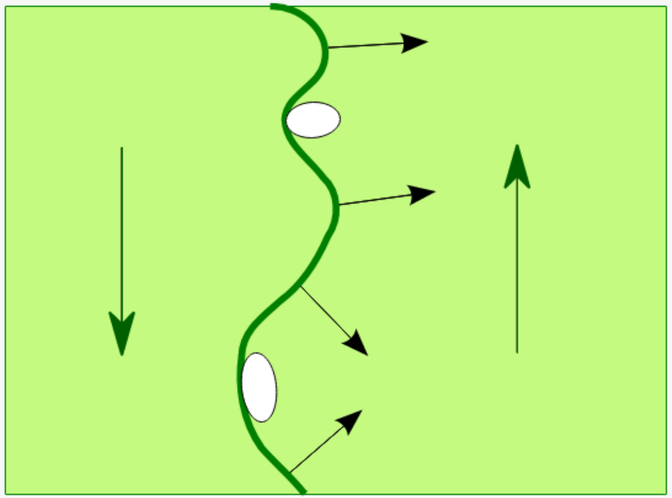
Coercivity determined by nucleation



- Concept of nucleation volume
- Physics has some similarity with that of the Stoner-Wohlfarth model for small particles



Coercivity determined by propagation



- Physics of surface/string in heterogeneous landscape
- Modeling necessary

Magnetostatics – Range considerations

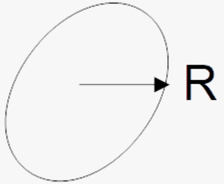
Range

Example: upper bound of dipolar field in thin films

Integration

$$\|\mathbf{H}_\mathrm{d}(\mathbf{r})\| \leq M_\mathrm{s} t \int \frac{2\pi r}{r^3}\,\mathrm{dr}$$

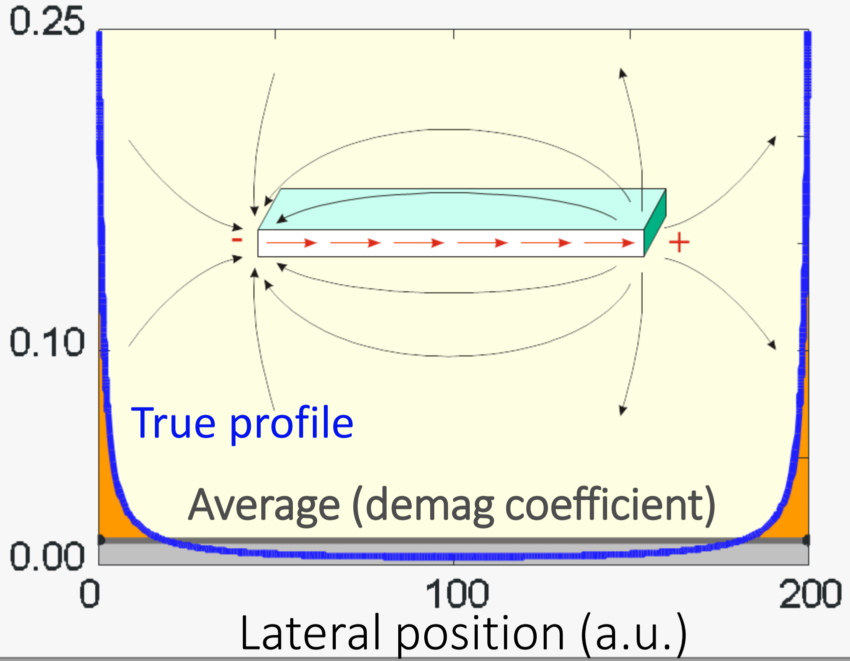
Hd for dipole



$$\|\mathbf{H}_\mathrm{d}(\mathbf{R})\| \leq \mathrm{Cste} + \mathcal{O}(1/R)$$

Non-homogeneity

Example: flat strip with aspect ratio 0.0125



- Dipolar fields are short-ranged and inhomogeneous in low dimensions
- Consequences: non-uniform magnetization switching, edge modes etc.

A 1D/2D system in space behaves very differently from a nano-bulk magnet

Magnetostatics – End domains and curling

Historical background

Introduced in the context of the Brown paradox for magnetization reversal

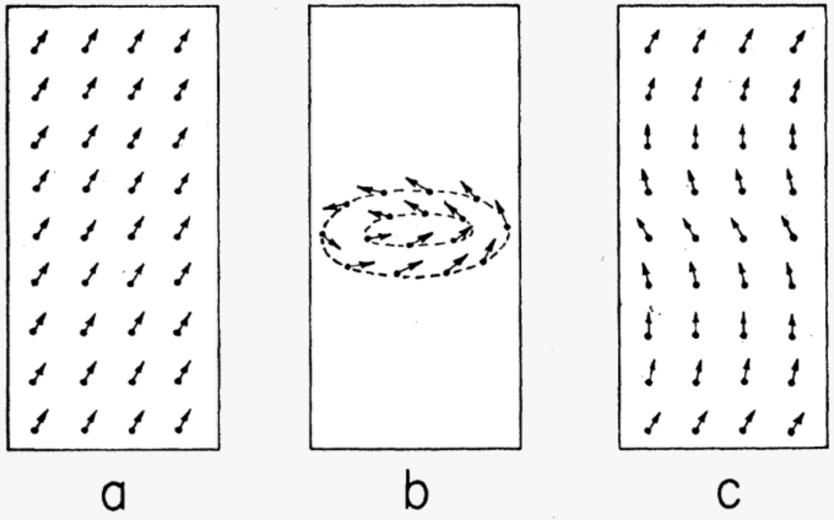


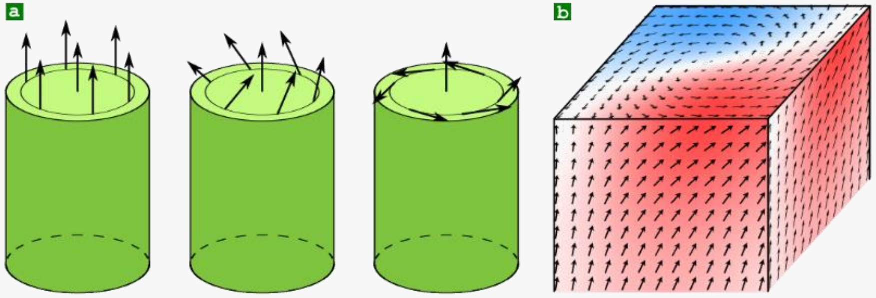
FIG. 2. Modes of magnetization change for the infinite cylinder: (a) spin rotation in unison; (b) magnetization curling; (c) magnetization buckling.

E. H. Frei, Phys. Rev. 106, 446 (1957)

Lecture
Manuel Vazquez

Example in 3D nanomagnets

End curling in elongated 3D objects (wires etc.)



Curling spreads surface charges into volume charges

$$\sigma(\mathbf{r}) = M_s\,\mathbf{m}(\mathbf{r})\cdot\mathbf{n}(\mathbf{r})$$

$$\rho(\mathbf{r}) = -M_s\,\boldsymbol{\nabla}\cdot\mathbf{m}(\mathbf{r}) = -M_s\frac{\partial m_z}{\partial z}$$

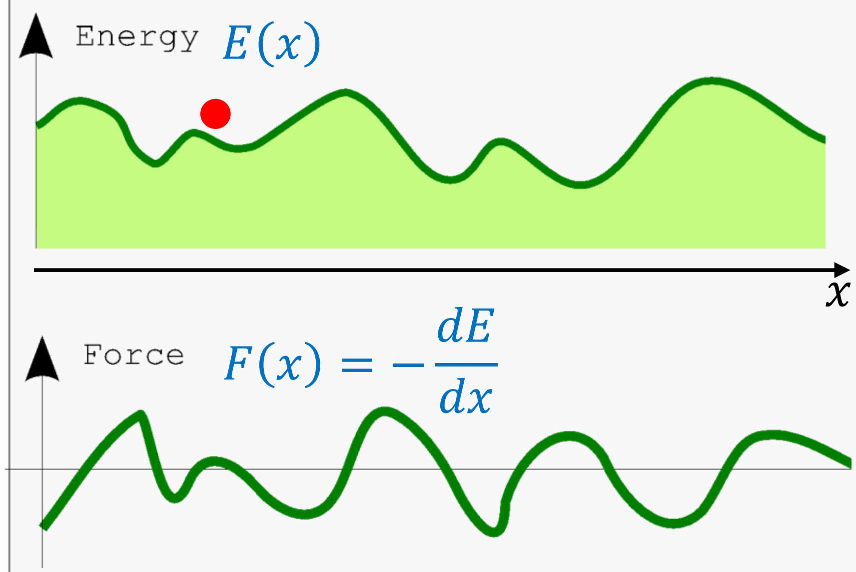
Notes

- Surface + volume charges is conserved
- Curling may develop whenever a dimension is larger than 7 dipolar exchange lengths $\Delta_d = \sqrt{2A/\mu_0 M_s^2}$

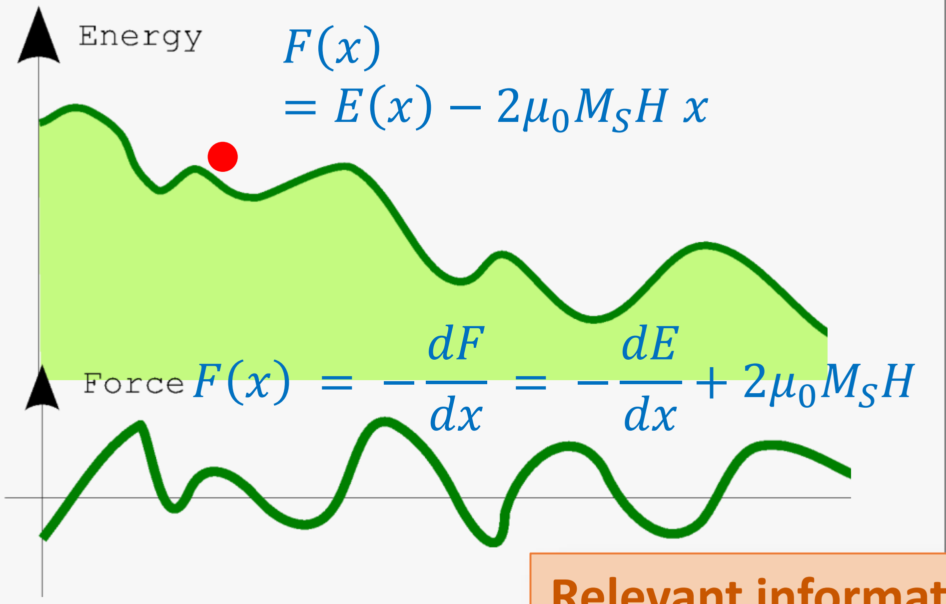
Pinning of domain walls

Example : domain wall to be moved along a 1d system

Without applied field



With applied field



E. Kondorski, On the nature of coercive force and irreversible changes in magnetisation, Phys. Z. Sowjetunion 11, 597 (1937)

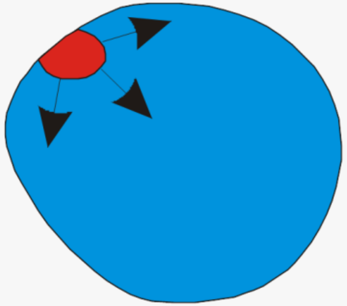
Relevant information

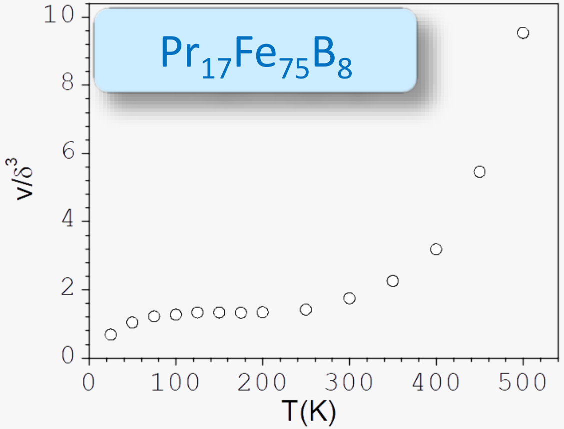
- Microstructure
- Chemical composition
- Crystal structure

Switching – Experiments

Activation volume

- Also called: nucleation volume
- Should be considered if system is larger than the characteristic length scale
- Use for: estimate $H_c(T)$, long-time relaxation, dimensionality
- Size similar to wall width δ





Courtesy D. Givord

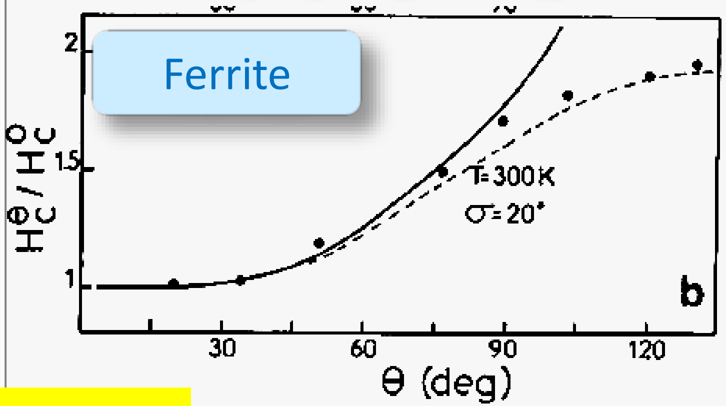
REVIEW: D. Givord et al., JMMM258, 1 (2003)

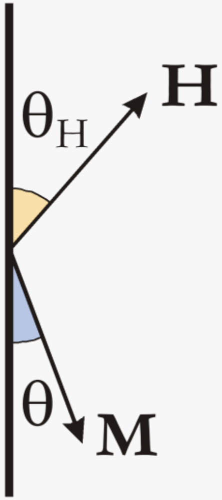
1/cos(ϑ) law, Becker-Kondorski model

E. J. Kondorsky, J. Exp. Theor. Fiz. 10, 420 (1940)

- Assumes: coercivity << anisotropy field
- Energy barriers overcome by Zeeman + thermal energy

$$\Delta E = -\mu_0 M_s H v_a \cos\theta_H + 25 k_B T$$



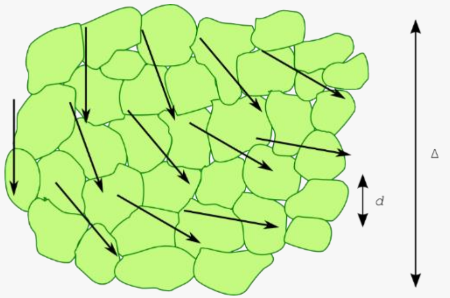


D. Givord et al., JMMM72, 247 (1988)

Lecture
Ester Palmero

Switching – Size dependence

Coercivity versus size of microstructure



$H_c \sim d^6$

G. Herzer, IEEE Trans. Magn 26, 1397 (1990)

Evanescent anisotropy

Superparamagnetism

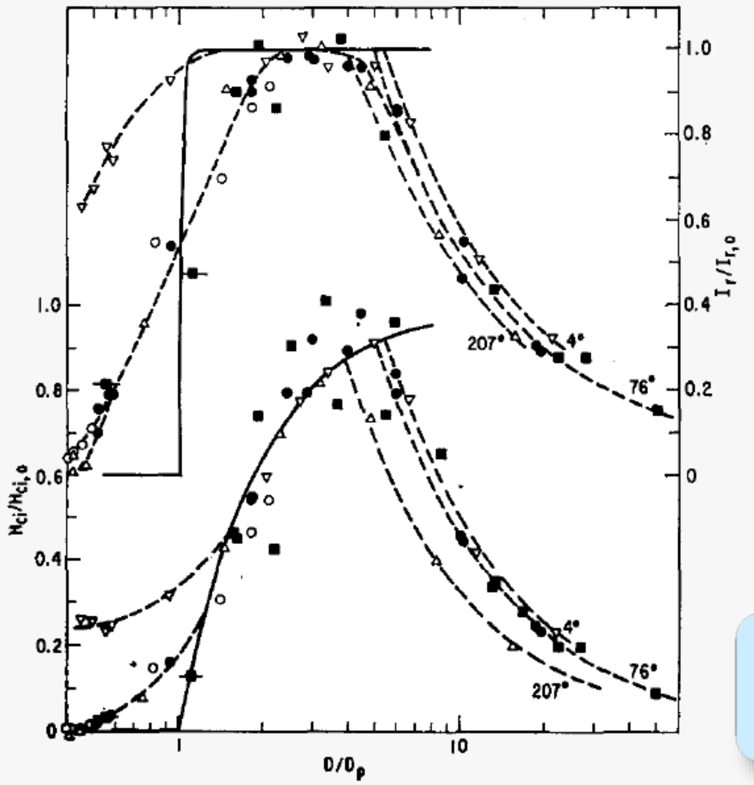
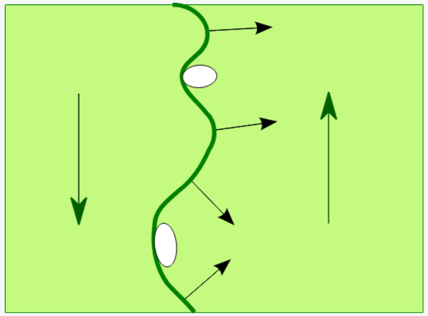


FIG. 1. Particle size dependence of essentially spherical, randomly oriented, iron particles. Calculated curve given by solid line. Diameters $D=\hat{d}_v$. Data at 76°K obtained from electron microscopic examination ■, calculated from I_r/I_s vs temperature O, and from smoothed data of H_{ci} vs D ●.



Towards nucleation-Propagation mechanisms

E. F. Kneller & F. E. Luborsky, Particle size dependence of coercivity and remanence of single-domain particles, J. Appl. Phys. 34, 656 (1963)

LLG equation

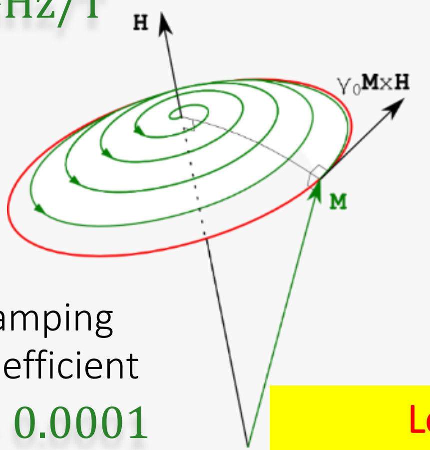
- Describes: precessional dynamics of magnetic moments
- Applies to magnetization, with phenomenological damping

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -|\gamma_0|\mathbf{m}\times\mathbf{H} + \alpha\,\mathbf{m}\times\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t}$$

$\gamma_0 = \mu_0\gamma < 0$ Gyromagnetic ratio

$\gamma_s = 28\ \mathrm{GHz/T}$

Larmor precession



$\alpha > 0$ Damping coefficient

$\alpha = 0.1 - 0.0001$

Ferromagnetic resonance

Small-angle precession

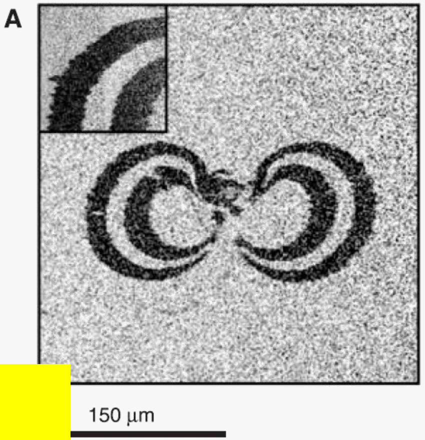

NATURE November 9, 1946 Vol. 158

Anomalous High-frequency Resistance of Ferromagnetic Metals

J. H. E. Griffiths, Anomalous high-frequency resistance of ferromagnetic materials, Nature, , 158, 670 (1946)

C. Kittel, Interpretation of Anomalous Larmor Frequencies in Ferromagnetic Resonance Experiment , Phys. Rev 71, 270 (1947)

Precessional switching of magnetization



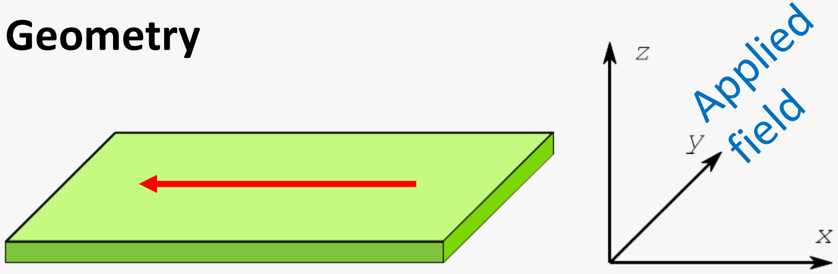
Large-angle precession

C. Back et al., Science 285, 864 (1999)

Lecture
Andrii Chumak

Precessional trajectories

Geometry

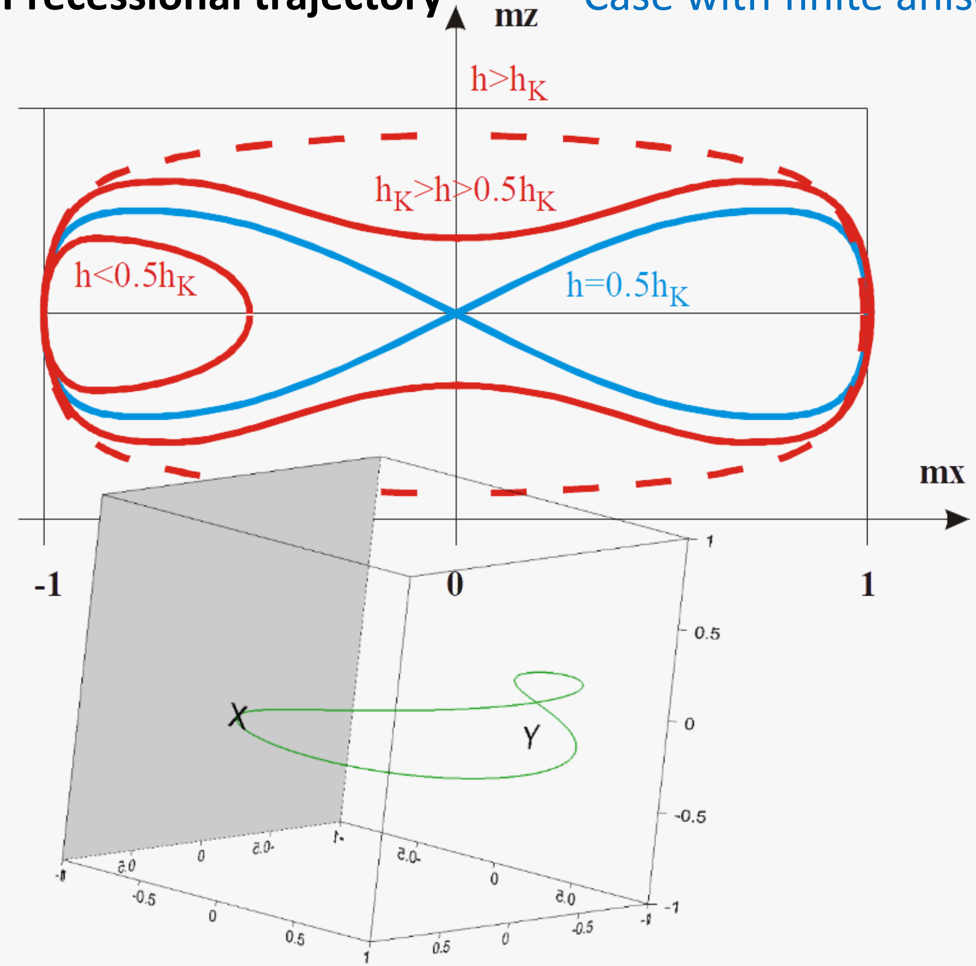


$$\frac{\mathbf{dm}}{\mathrm{d}t} = -|\gamma_0|\mathbf{m} \times \mathbf{H} + \text{damping}$$

- Precession around its own demagnetizing field
- Threshold for switching is half the Stoner-Wohlfarth one

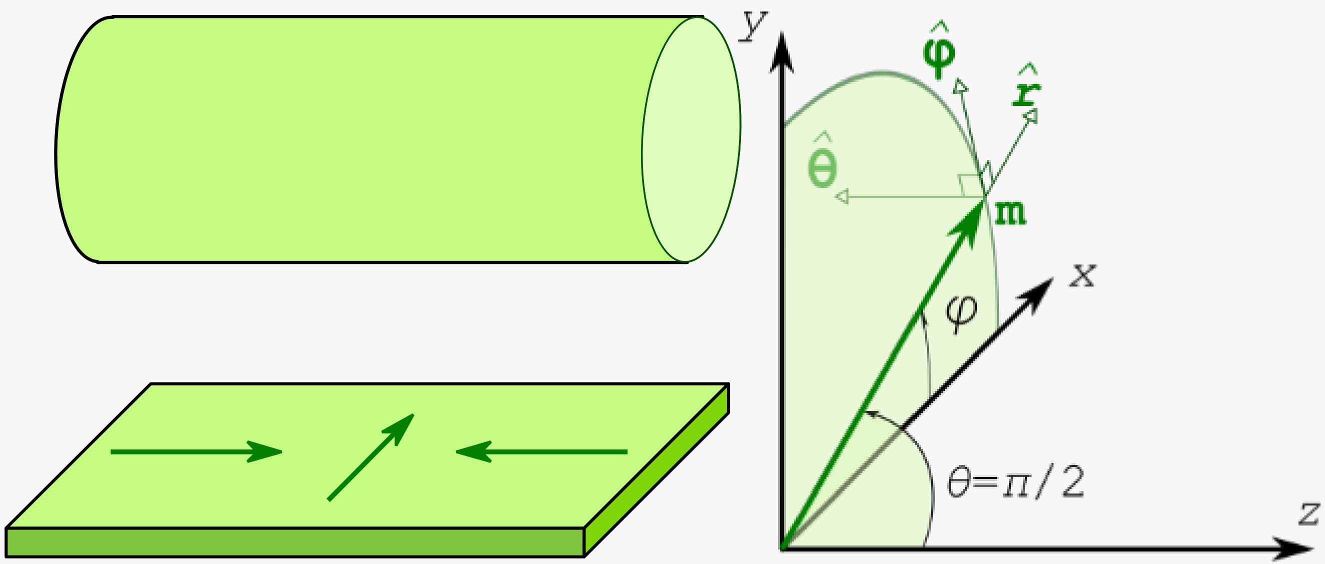
Precessional trajectory

Case with finite anisotropy



Precessional motion of magnetic domain walls

Precessional dynamics of transverse walls under magnetic field

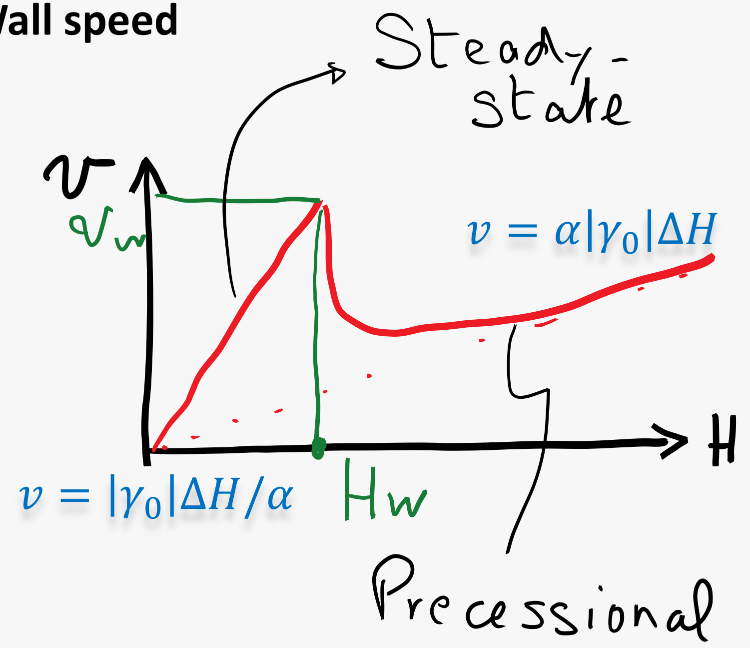


Precessional dynamics under magnetic field

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = -|\gamma_0|\mathbf{m}\times\mathbf{H} + \alpha\,\mathbf{m}\times\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t}$$

A. Thiaville, Y. Nakatani, Domain-wall dynamics in nanowires and nanostrips, in *Spin dynamics in confined magnetic structures {III}*, Springer (2006)

Wall speed



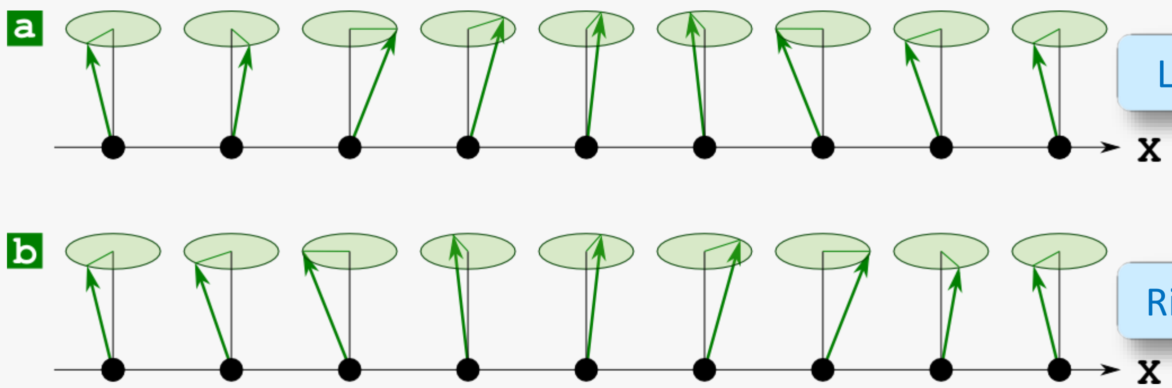
- Walker field $H_W = \alpha M_s/2$ $\approx$ few mT
- Walker speed $v = |\gamma_0| M_s \Delta/2$

up to ≈ 100 m/s, to km/s

Spin waves

Propagating Larmor precession

- **Physics**: exchange promotes propagation
- Spin waves have an angular frequency ω and a vector for propagation
- **There exist various geometries**, related to the direction of $\mathbf{M}$ versus $\mathbf{k}$, and the geometry of the system (thin film etc.)



Left-propagating

Right-propagating

Dispersion curve

- **Physics**: exchange implies additional energy, and thus higher frequency

$$\omega(k) = \omega_0 + Dk^2$$

D Spin-wave stiffness coefficient

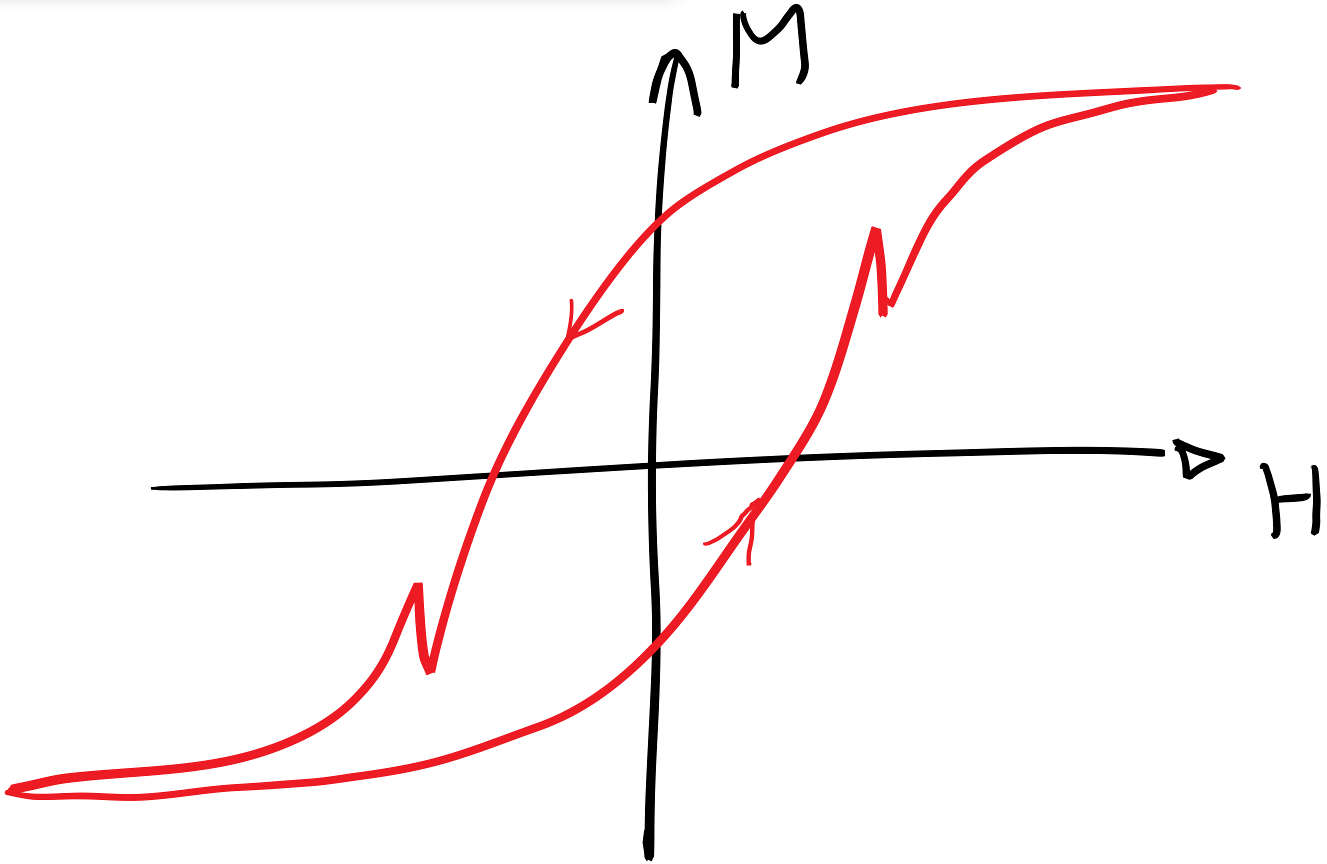
- **Dipolar energy**: depending on the spin-wave geometry, dipolar energy provides additional contributions to D, possibly with a negative value.

Situations for spin waves

- **Thermally-excited** → Contributes to the decay of magnetization with temperature
- **Magnonics:** excited on purpose using a radio-frequency field or a spin-polarized current.

Quizz #2

Is such a hysteresis loop possible ?




"That's all Folks!"