Units (and quantities) in Magnetism

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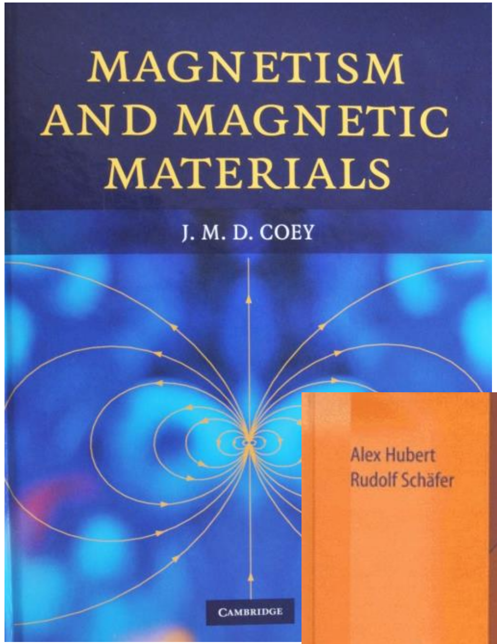
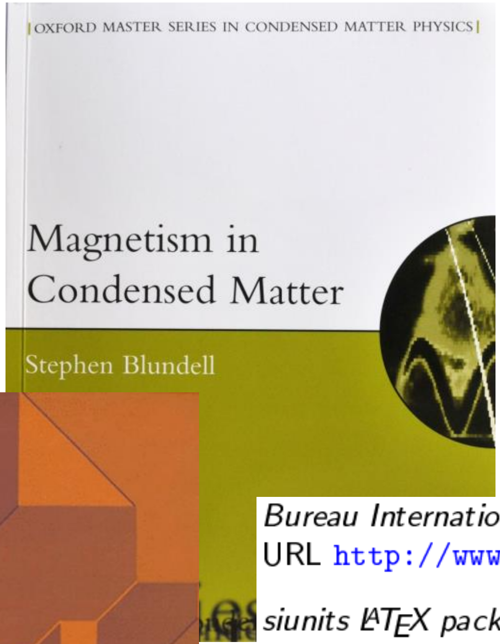
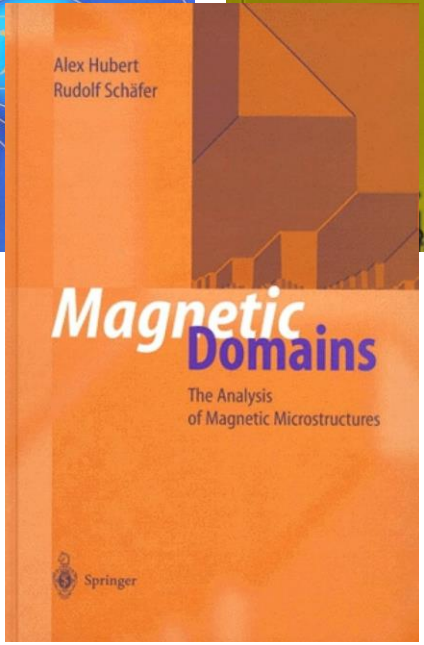
Slides: http://fruchart.eu/slides

Some references







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What is a quantity?

What is a unit ?

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Quantity

- Example: speed $\mathbf{v} = \delta\boldsymbol{\ell}/\delta t$
- Dimension: $\dim(\mathbf{v}) = \mathrm{L}\cdot\mathrm{T}^{-1}$

Units

- Why?
  - Provide a measure
  - Universality: share with others
- Possible formalism:

$$X = X_\alpha \langle X \rangle_\alpha$$

Quantity

Measure

Reference quantity

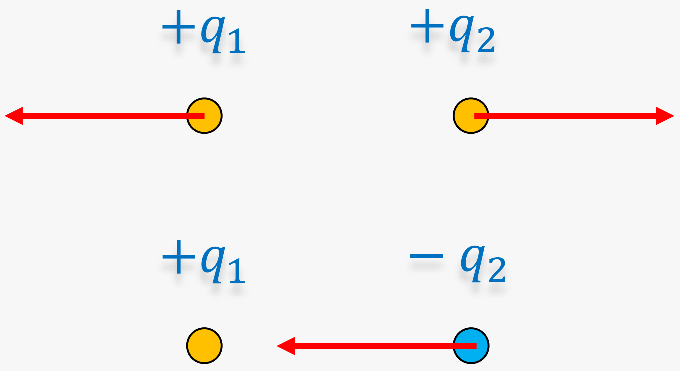
$$\langle L \rangle_{\mathrm{SI}} = \mathrm{meter} = 100 \langle L \rangle_{\mathrm{cgs}}$$

$$L = 50 \langle L \rangle_{\mathrm{SI}} = 5000 \langle L \rangle_{\mathrm{cgs}}$$

The electric charge and the electric field

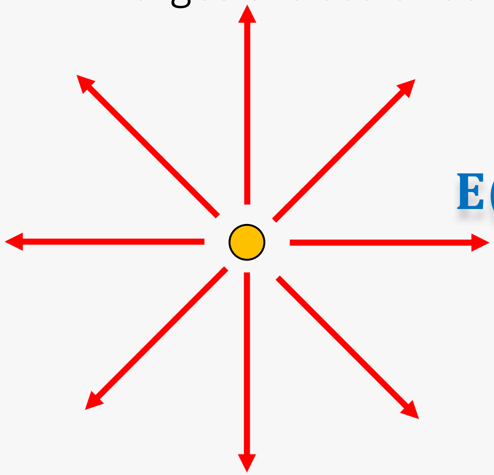
Facts: force between charges

$$\mathbf{F}_{1\to2} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\mathbf{u}_{12}$$



Modeling by the Physicist

- Electric field $\mathbf{E}_{1\to2}$ $\qquad \mathbf{F}_{1\to2} = \mathrm{q}_2\mathbf{E}_{1\to2}$
- Charges are scalar sources of electric field

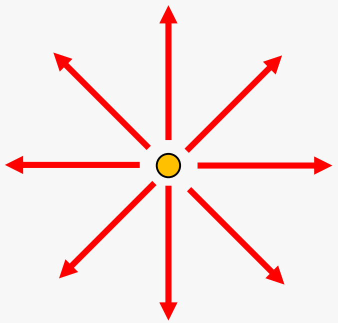


$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0 r^2}\mathbf{u}$$

The electric charge and the electric field

Macroscopic level: Gauss theorem

- Ostogradski theorem



$$\iiint_{\mathcal{V}} \boldsymbol{\nabla}\cdot\mathbf{E}\,\mathrm{d}\mathcal{V} = \oiint_{\partial\mathcal{V}} \mathbf{E}\cdot\mathbf{n}\,\mathrm{d}\mathcal{S}$$

$$\Rightarrow \quad \frac{Q}{\epsilon_0} = \iiint_{\mathcal{V}} \frac{\rho}{\epsilon_0}\,\mathrm{d}\mathcal{V} = \oiint_{\partial\mathcal{V}} \mathbf{E}\cdot\mathbf{n}\,\mathrm{d}\mathcal{S}$$

Link

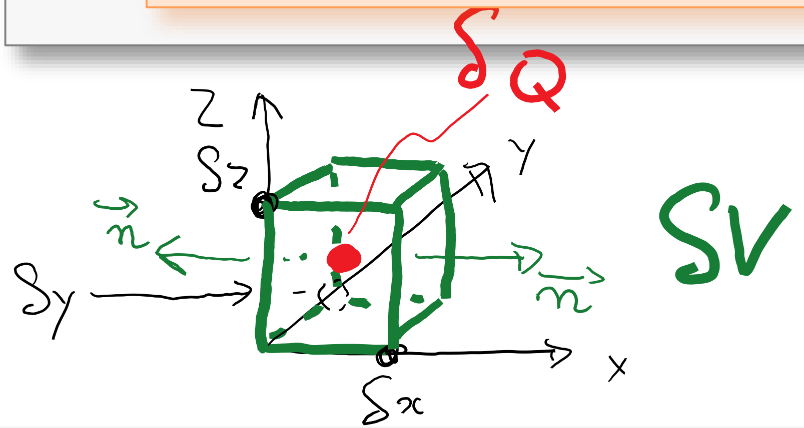
$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{\partial E_x}{\partial x} + \ldots = \frac{E_x(x+\delta x) - E_x(x)}{\delta x} + \cdots$$

Microscopic level: Maxwell equation

$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\rho = \frac{\delta Q}{\delta\mathcal{V}}$$ Volume density of electric charge

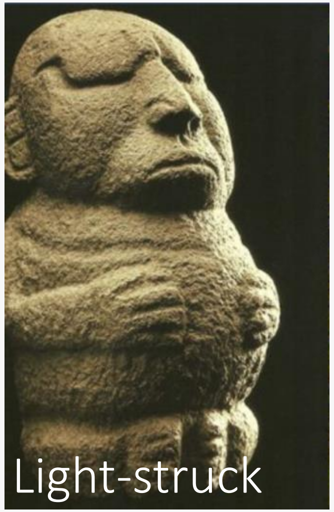
- *Q* is the scalar source of **E**



Origin of magnetism

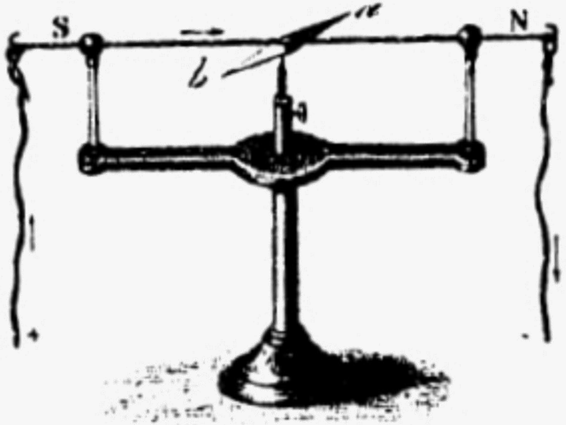
Century-old facts

- Magnetic materials (rocks)



- Magnetic field of the earth

Œrsted experiment in 1820

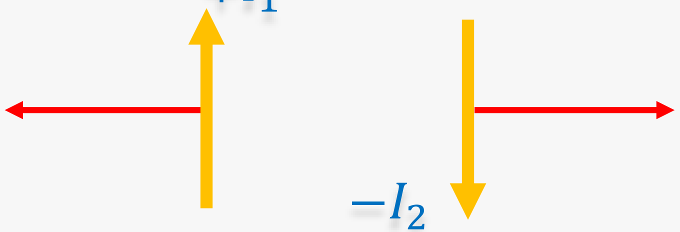


Hans-Christian Oersted, 1777–1851.

Birth of electromagnetism

Facts: force between charge currents

$$\delta\mathbf{F}_{1\to2} = \mu_0 \frac{I_1 I_2[\delta\boldsymbol{\ell}_2 \times (\delta\boldsymbol{\ell}_1 \times \mathbf{u}_{12})]}{4\pi r_{12}^2}$$

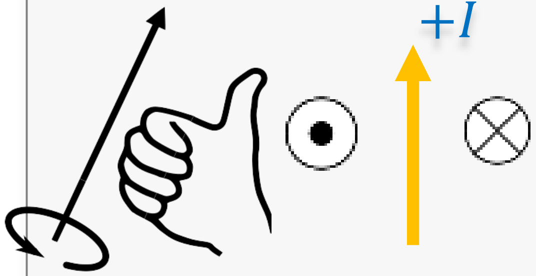


<u>Note</u>: former definition of the Ampère:

The force between two infinite wires 1m apart with current 1A ~~is~~ was $2\times10^{-7}\ \mathrm{N/m}$

Modeling by the Physicist

- Magnetic induction field: Biot & Savart law



$$\delta\mathbf{B} = \frac{\mu_0 I \delta\boldsymbol{\ell} \times \mathbf{u}}{4\pi r^2}$$

- Retrieve the force (Laplace)

$$\delta\mathbf{F}_2 = I_2 \delta\boldsymbol{\ell} \times \mathbf{B}(\mathbf{r}_2)$$

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

- Magnetic induction field defined through Lorentz Force

Macroscopic level: Ampere theorem

- Stokes theorem

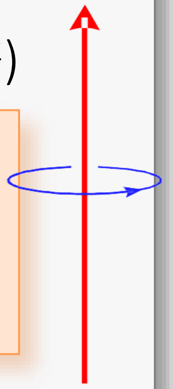
$$\iint_{\mathcal{S}} (\boldsymbol{\nabla}\times\mathbf{B})\cdot\mathbf{n}\,\mathrm{d}\mathcal{S} = \oint_{\partial\mathcal{S}} \mathbf{B}\cdot\mathrm{d}\boldsymbol{\ell}$$

$$\Rightarrow \quad I = \mu_0 \iint_{\mathcal{S}} (\mathbf{j}\cdot\mathbf{n})\,\mathrm{d}\mathcal{S} = \oint_{\partial\mathcal{S}} \mathbf{B}\cdot\mathrm{d}\boldsymbol{\ell}$$

Microscopic level: Maxwell equation

$$\boldsymbol{\nabla}\times\mathbf{B} = \mu_0\mathbf{j}$$

j: Volume density of current (A/m^2)

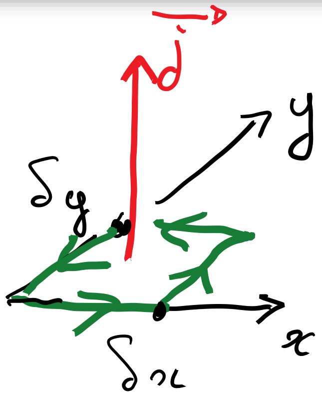


- j is the vectorial source of curl of **B**

Unit for B: tesla (T)

Link

$$\boldsymbol{\nabla}\times\mathbf{B} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{\partial B_y}{\partial x} - \dfrac{\partial B_x}{\partial y} \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{B_y(x+\delta x) - B_y(x)}{\delta x} - \dfrac{B_x(y+\delta y) - B_x(y)}{\partial y} \end{pmatrix}$$



Maxwell equations (in vacuum)

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$ → Gauss theorem

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ → Faraday law of induction

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$ → Ampère theorem

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0$$ → B is divergence free
(no magnetic poles)

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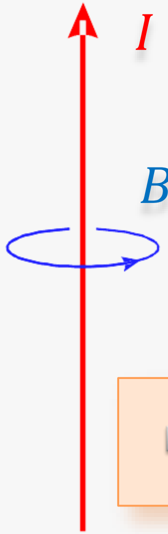
Biot and Savart

$$\delta\mathbf{B} = \frac{\mu_0 I \delta\boldsymbol{\ell} \times \mathbf{u}}{4\pi r^2}$$

- Note: $1/r^2$ decay

Ampere theorem and Œrsted field

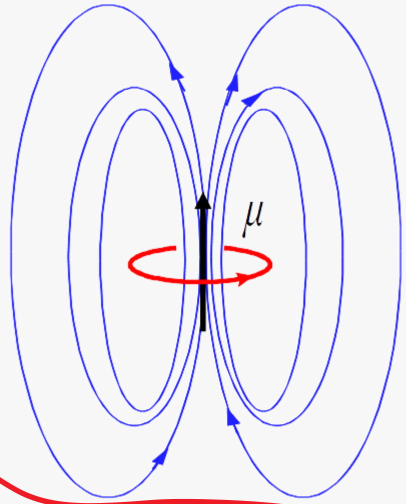


$$B_\theta = \frac{\mu_0 I}{2\pi r}$$

Integrate

- Note: $1/r$ decay

The magnetic point dipole



- Simple loop

$\boldsymbol{\mu} = I\mathcal{S}\,\mathbf{n}$ **Unit**: $\mathrm{A}\cdot\mathrm{m}^2$

- General definition

$$\boldsymbol{\mu} = \frac{1}{2}\iiint_{\mathcal{V}} \mathbf{r}\times\mathbf{j}(\mathbf{r})\,\mathrm{d}\mathcal{V}$$

Derive

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left[\frac{3}{r^2}(\boldsymbol{\mu}\cdot\mathbf{r})\mathbf{r} - \boldsymbol{\mu}\right]$$

- Note: $1/r^3$ decay

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}(2\mu\cos\theta\,\mathbf{u}_r + \mu\sin\theta\,\mathbf{u}_\theta)$$

Magnetization

Definition

- Volume density of magnetic point dipoles

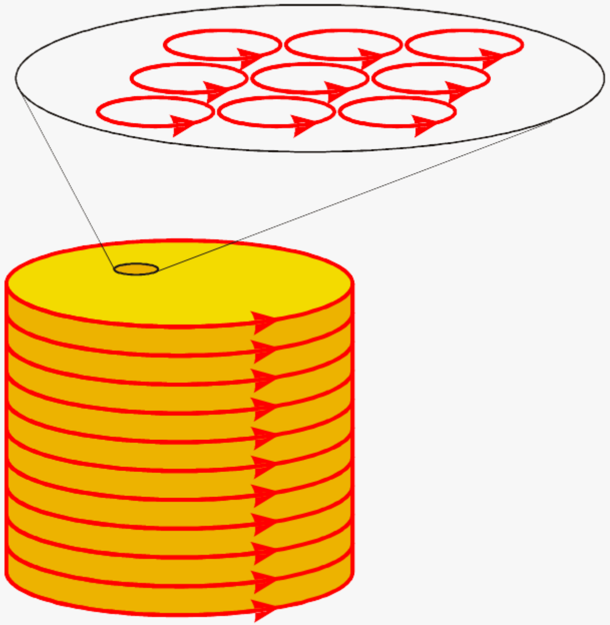
$$\mathbf{M} = \frac{\delta \boldsymbol{\mu}}{\delta \mathcal{V}} \qquad \mathrm{A/m}$$

- Total magnetic moment of a body

$$\boldsymbol{\mathcal{M}} = \int_{\mathcal{V}} \mathbf{M}\,\mathrm{d}\mathcal{V} \qquad \mathrm{A \cdot m^2}$$

- Applies to: ferromagnets, paramagnets, diamagnets etc.
- Must be defined at a length scale much larger than atoms
- Is the basis for the micromagnetic theory

Equivalence with surface currents



- Name: Amperian description of magnetism
- Surface current equals magnetization $\mathrm{A/m}$

Free currents and bound currents

Back to Maxwell equations

- Disregard fast time dependence: magnetostatics

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \cancel{\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}} \right)$$

- Consider separately real charge current, $\mathbf{j}_\mathrm{c}$ from fictitious currents of magnetic dipoles $\mathbf{j}_\mathrm{m}$

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{j}_\mathrm{c} + \mathbf{j}_\mathrm{m})$$

- One can show: $\nabla \times \mathbf{M} = \mathbf{j}_\mathrm{m}$ $\quad \mathrm{A/m^2}$

$\mathbf{M} \times \mathbf{n} = \mathbf{j}_\mathrm{m,s}$ $\quad \mathrm{A/m}$

The magnetic field H

- One has: $\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{j}_\mathrm{c}$

- By definition: $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ $\quad \mathrm{A/m}$

$$\nabla \times \mathbf{H} = \mathbf{j}_\mathrm{c}$$

B versus H : definition of the system

- **M**: local (infinitesimal) part in $\delta\mathcal{V}$ of the system defined when considering a magnetic material

- **H**: The remaining of B coming from outside $\delta\mathcal{V}$, liable to interact with the system

- Outside matter, $\mathbf{B}$ and $\mu_0\mathbf{H}$ coincide and have exactly the same meaning.

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The dipolar field H_d

- By definition: the contribution to **H** not related to free currents (possible to split as Maxwell equations are linear)

$$\nabla \times \mathbf{H}_\mathrm{d} = 0 \;\Rightarrow\; \mathbf{H}_\mathrm{d} = -\nabla \phi_\mathrm{d}$$

$$\mathbf{H} = \mathbf{H}_\mathrm{d} + \mathbf{H}_\mathrm{app}$$

External to magnetic body

Analogy with electrostatics

$$\nabla \times \mathbf{E} = 0 \;\Rightarrow\; \mathbf{E} = -\nabla \phi$$

Derive the dipolar field

Maxwell equation $\nabla \cdot \mathbf{B} = 0 \;\rightarrow\; \nabla \cdot \mathbf{H}_\mathrm{d} = -\nabla \cdot \mathbf{M}$

$$\Rightarrow \mathbf{H}_\mathrm{d}(\mathbf{r}) = -M_\mathrm{s} \iiint_{\mathcal{V}'} \frac{[\nabla \cdot \mathbf{m}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{V}'$$

To lift the singularity that may arise at boundaries, a volume integration around the boundaries yields:

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \iiint \frac{\rho(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{V}' + \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{S}'$$

$\rho(\mathbf{r}) = -M_\mathrm{s}\,\nabla \cdot \mathbf{m}(\mathbf{r})$ → volume density of magnetic charges

$\sigma(\mathbf{r}) = M_\mathrm{s}\,\mathbf{m}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r})$ → surface density of magnetic charges

Stray field and demagnetizing field

Vocabulary

- Generic names

  Magnetostatic field

  Dipolar field

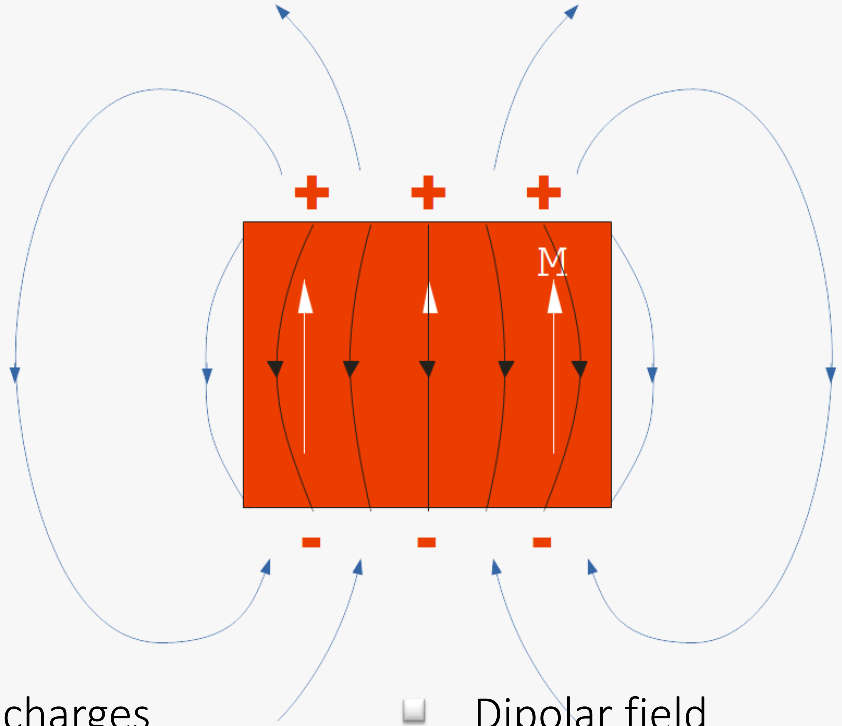
- Inside material

  Demagnetizing field

- Outside material

  Stray field

Example Permanent magnet (uniformly-magnetized)



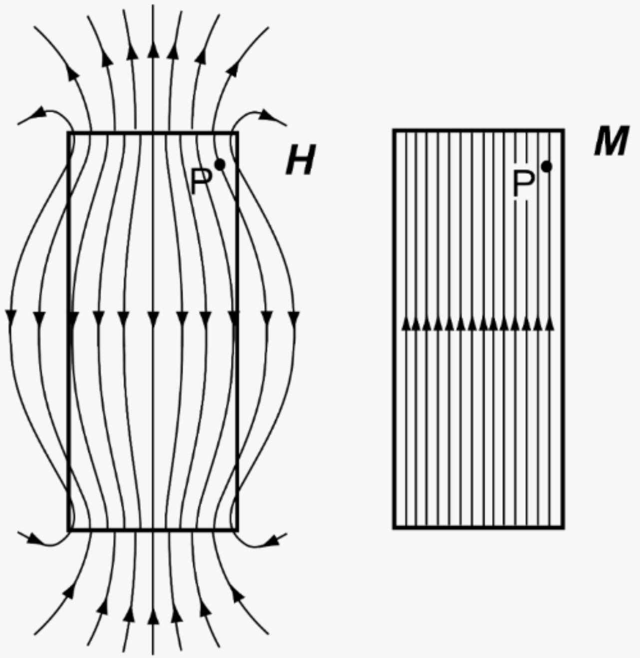
- Surface charges

$$\sigma(\mathbf{r}) = M_s \mathbf{m}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r})$$

- Dipolar field

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{S}'$$

Example Permanent magnet (uniformly-magnetized)



- Surface charges

$$\sigma(\mathbf{r}) = M_s \mathbf{m}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r})$$

- Dipolar field

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{S}'$$

Vocabulary

- Generic names
  - Magnetostatic field
  - Dipolar field
- Inside material
  - Demagnetizing field
- Outside material
  - Stray field

Illustration from: M. Coey's book

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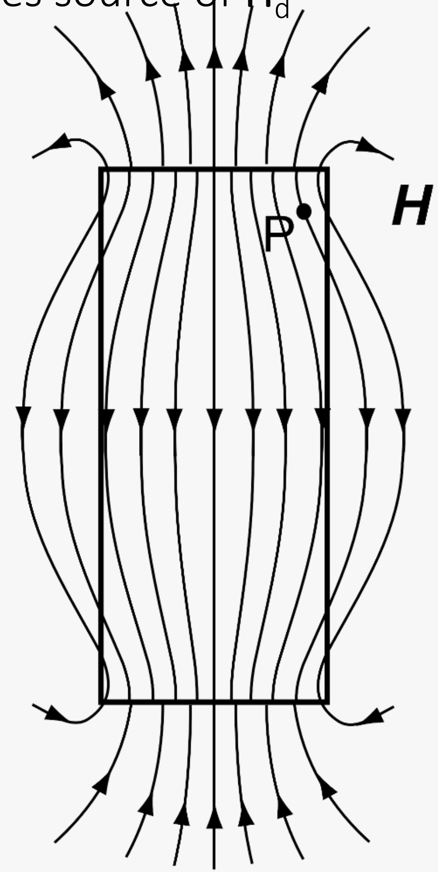
Coulombian

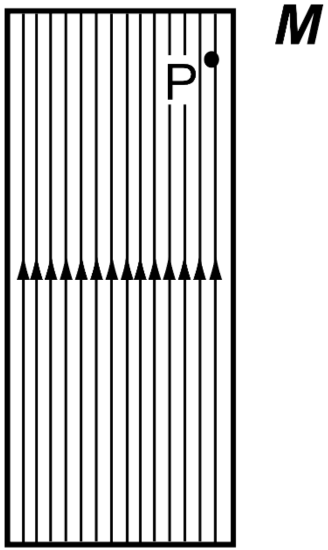
- Pseudo-charges source of H_d



$$\nabla \times \mathbf{H} = \mathbf{0}$$

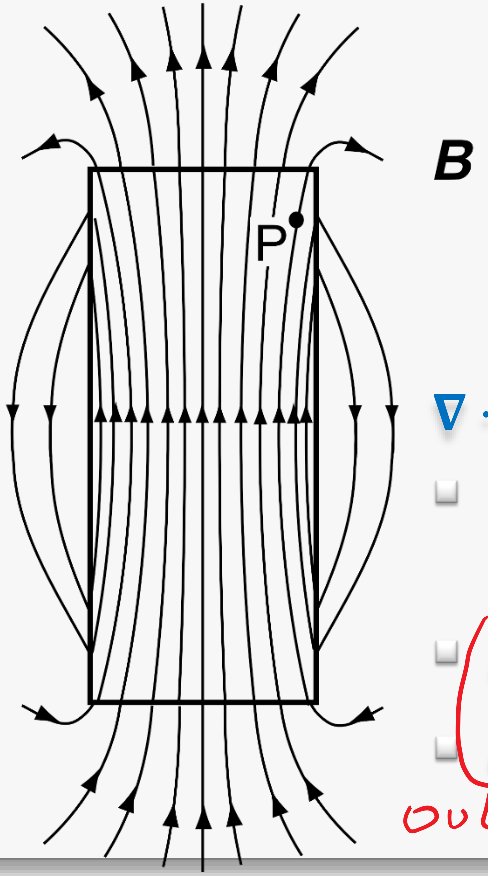
- No closed lines
- $\Delta H_{\parallel} = 0$
- $\Delta \mathbf{H} \cdot \mathbf{n} = \sigma$

out - in



Amperian

- Fictitious currents source of B



$$\nabla \cdot \mathbf{B} = \mathbf{0}$$

- No magnetic monopole
- $\Delta B_{\perp} = 0$
- $\Delta \mathbf{B} = \mu_0 \mathbf{j} \times \mathbf{n}$

out - in

From: M. Coey's book

Dimensional analysis (example: lengths)

The dipolar exchange length

When: anisotropy and exchange compete

$$E = A\left(\frac{\partial m_i}{\partial x_j}\right)^2 + K_\mathrm{d}\sin^2\theta$$

Exchange: $\mathrm{J/m}$ — Dipolar: $\mathrm{J/m^3}$

$$K_\mathrm{d} = \frac{1}{2}\mu_0 M_\mathrm{s}^2$$

$$\boxed{\Delta_\mathrm{d} = \sqrt{A/K_\mathrm{d}} = \sqrt{2A/\mu_0 M_\mathrm{s}^2}}$$

$$\Delta_\mathrm{d} \simeq 3 - 10\ \mathrm{nm}$$

Critical single-domain size, relevant for small particles made of soft magnetic materials

Often called: exchange length

The anisotropy exchange length

When: anisotropy and exchange compete

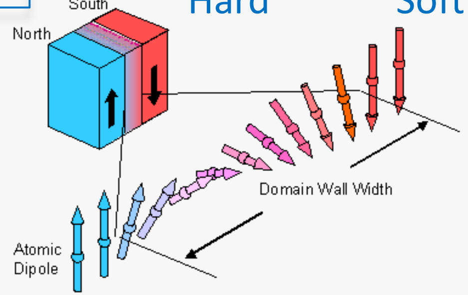
$$E = A\left(\frac{\partial m_i}{\partial x_j}\right)^2 + K\sin^2\theta$$

Exchange: $\mathrm{J/m}$ — Anisotropy: $\mathrm{J/m^3}$

$$\boxed{\Delta_\mathrm{u} = \sqrt{A/K}}$$

$$\Delta_\mathrm{u} \simeq 1\ \mathrm{nm} \rightarrow 100\ \mathrm{nm}$$

Hard — Soft



Sometimes called: Bloch parameter, or wall width

Note: Other length scales can be defined, e.g. with magnetic field

Units in SI versus cgs

Definitions	S.I.		cgs-Gauss	
	Meter	m	Centimeter	cm
	Kilogram	kg	Gram	g
	Second	s	Second	s
	Ampere	A	Ab-Ampere	ab-A = 10 A
	$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$		$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$	
	$\mu_0 = 4\pi \times 10^{-7}$ S. I.		$"\mu_0" = 4\pi.$	

Problems with cgs

- The quantity for charge current is missing
  No check for homogeneity;
  paradox for spintronics
- Inconsistent definition of H
  Dimensionless quantities are affected:
  demag coefficients, susceptibility etc.

Conversion of measures for the same quantity

Field	$\mathbf{H}$	$1\ \mathrm{A/m}$	$\longleftrightarrow$	$4\pi \times 10^{-3}\ \mathrm{Oe}$	Œrsted
Moment	$\boldsymbol{\mu}$	$1\ \mathrm{A \cdot m^2}$	$\longleftrightarrow$	$10^3\ \mathrm{emu}$	
Magnetization	$\mathbf{M}$	$1\ \mathrm{A/m}$	$\longleftrightarrow$	$10^{-3}\ \mathrm{emu/cm^3}$	Electromagnetic Unit
Induction	$\mathbf{B}$	$1\ \mathrm{T}$	$\longleftrightarrow$	$10^4\ \mathrm{G}$	Gauss
Susceptibility	$\chi = M/H$	1	$\longleftrightarrow$	$1/4\pi$	

Tutorial on units
Questions: http://magnetism.eu/esm/2018/abs/fruchart-practical-abs1.pdf
Answers: http://magnetism.eu/esm/2018/abs/fruchart-practical-answers1.pdf

How to convert units from one system to another?

Example: length $X = X_\alpha \langle X \rangle_\alpha$

Reference quantity

Quantity

Measure

The SI standard is 100 times **LARGER** than the cgs one

$$L = 50\langle L \rangle_{\mathrm{SI}} = 5000\langle L \rangle_{\mathrm{cgs}}$$

$50\ \mathrm{m}$ is equivalent to $5000\ cm$

The SI measure is 100 times **SMALLER** than the cgs one

The ratio is opposite if one considers the standard for a quantity (a quantity) or the measure (a number) of a given quantity

How to convert units from one system to another?

Process for converting units

1. Convert all basic units (MKSA)

$$\langle \mathrm{L} \rangle_{\mathrm{SI}} = \mathrm{meter} = 10^2 \langle \mathrm{L} \rangle_{\mathrm{cgs}}$$

$$\langle \mathrm{M} \rangle_{\mathrm{SI}} = \mathrm{kilogram} = 10^3 \langle \mathrm{M} \rangle_{\mathrm{cgs}}$$

$$\langle \mathrm{T} \rangle_{\mathrm{SI}} = \mathrm{second} = \langle \mathrm{T} \rangle_{\mathrm{cgs}}$$

$$\langle \mathrm{I} \rangle_{\mathrm{SI}} = \mathrm{Ampère} = 10^{-1} \langle \mathrm{I} \rangle_{\mathrm{cgs}}$$

2. Decompose any given quantity in basics units. In practice, identify a formula linking if to quantities already decomposed
3. Apply the formalism defining units and measures

$$X = X_\alpha \langle X \rangle_\alpha$$

Example Mechanics, force F

$$\mathbf{F} = m\,\mathbf{a}$$

$$\dim(\mathbf{F}) = \mathrm{M} \cdot \mathrm{L} \cdot \mathrm{T}^{-2}$$

$$\begin{aligned} F &= F_{\mathrm{SI}} \langle F \rangle_{\mathrm{SI}} \\ &= F_{\mathrm{SI}}\, \langle \mathrm{L} \rangle_{\mathrm{SI}}\, \langle \mathrm{M} \rangle_{\mathrm{SI}}\, \langle \mathrm{T} \rangle_{\mathrm{SI}}^{-2} \\ &= F_{\mathrm{SI}}\, 10^2 \langle \mathrm{L} \rangle_{\mathrm{cgs}}\, 10^3 \langle \mathrm{M} \rangle_{\mathrm{cgs}}\, (1)^{-2} \langle \mathrm{T} \rangle_{\mathrm{cgs}}^{-2} \\ &= F_{\mathrm{SI}}\, 10^5 \langle \mathrm{F} \rangle_{\mathrm{cgs}} \end{aligned}$$

$1\ \mathrm{N}$ is equivalent to $10^5\ \mathrm{erg}$

How to convert units from one system to another?

Proposed logarithmic formalism for dimensionality

$$\dim(\mathbf{X}) = \mathrm{L}^{\alpha} \cdot \mathrm{M}^{\beta} \cdot \mathrm{T}^{\gamma} \cdot \mathrm{I}^{\delta}$$

		$\langle X \rangle_{\mathrm{SI}} / \langle X \rangle_{\mathrm{cgs}}$	Log
M (meter)	$[\mathrm{L}] = [1\ 0\ 0\ 0]$	10^2	2
K (kg)	$[\mathrm{M}] = [0\ 1\ 0\ 0]$	10^3	3
S (second)	$[\mathrm{T}] = [0\ 0\ 1\ 0]$	1	0
A (Ampère)	$[\mathrm{I}] = [0\ 0\ 0\ 1]$	10^{-1}	-1

$$[\mathbf{X}] = \alpha[\mathrm{L}] + \beta[\mathrm{M}] + \gamma[\mathrm{T}] + \delta[\mathrm{I}]$$

$$[\mathbf{X}] = [\alpha\ \beta\ \delta\ \gamma]$$

Example Mechanics, force F

$$\mathbf{F} = m\,\mathbf{a}$$

$$[\mathbf{F}] = [m] + [\mathbf{a}] = [0\ 1\ 0\ 0] + [1\ 0\ -2\ 0]$$

$$[\mathbf{F}] = [1\ 1\ -2\ 0]$$

$$2\ \ 3\ \ \ \ 0$$

$$2\ \ 3\ \ \ \ 0\ \ \ \ \ \ \to 5$$

$1\ \mathrm{N}$ is equivalent to $10^5\ \mathrm{erg}$

Dimensionality of units in magnetism

Dimensionality

- A magnetic moment has the dimension of a pinpoint magnetic dipole $\boldsymbol{\mu} = I\mathbf{S}$. thus, $[\boldsymbol{\mu}] = [2\;0\;0\;1]$.
- Magnetization is a volume density of magnetic moments: $\mathbf{M} = \boldsymbol{\mu}/V$, so: $[\mathbf{M}] = [-1\;0\;0\;1]$. $\mathbf{M}$ and $\mathbf{H}$ have the same dimension as we can see from: $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$. Thus: $[\mathbf{H}] = [-1\;0\;0\;1]$.
- Magnetic induction B is what matters in Lorentz force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, so that: $[\mathbf{B}] = [0\;1\;-2\;-1]$.
- Magnetic flux is $\phi = BS$ so that: $[\phi] = [2\;1\;-2\;-1]$.
- Finally, as in electricity, μ_0 makes the link between the source (current) and fields on one side, and energy and mechanics on the other side, as for the Lorentz force above: $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, or in vacuum: $\mathbf{curl}\,\mathbf{B} = \mu_0\mathbf{j}$, from which one derives: $[\mu_0] = [1\;1\;-2\;-2]$.

Units (easy situations)

- Induction $\mathbf{B}$. $1\,\mathrm{T}$ is equivalent to $10^4\,\mathrm{G}$, G standing for *Gauss*.
- Magnetization $\mathbf{M}$. $1\,\mathrm{A/m}$ is equivalent to $10^{-3}\,\mathrm{uem/cm^3}$, emu standing for *ElectroMagnetic Unit*.
- Flux ϕ. $1\,\mathrm{Wb}$ (Weber) is equivalent to $10^8\,\mathrm{Mx}$, Mx standing for *Maxwell*.
- Moment $\boldsymbol{\mu}$. $1\,\mathrm{A\cdot m^2}$ is equivalent to $10^3\,\mathrm{emu}$.

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Tricky case 1: magnetic permeability

$$\mu_0 = \mu_{0\,\mathrm{SI}}\langle\mu_0\rangle_{\mathrm{SI}}$$

$$[\mu_0] = [1\ 1\ -2\ -2]$$

$$\rightarrow\ \langle\mu_0\rangle_{\mathrm{SI}} = 10^2\cdot 10^3\cdot(10^{-2})^{-1}\langle\mu_0\rangle_{\mathrm{cgs}}$$

$$\rightarrow\ \langle\mu_0\rangle_{\mathrm{SI}} = 10^7\langle\mu_0\rangle_{\mathrm{cgs}}$$

$$\mu_0 = \mu_{0\,\mathrm{SI}}\langle\mu_0\rangle_{\mathrm{SI}} \rightarrow \text{"}\mu_{0_{\mathrm{cgs}}}\text{"} = 4\pi$$

S.I. $\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$

cgs-Gauss $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

→ Unit for permeability dropped; H 4pi larger in cgs

Tricky case 2: magnetic field H

SI: $\mu_0 H = \mu_{0\,\mathrm{SI}}\langle\mu_0\rangle_{\mathrm{SI}}\ H_{\mathrm{SI}}\langle H\rangle_{\mathrm{SI}}$

$$\mu_0 H = 4\pi 10^{-7}\ 10^7\langle\mu_0\rangle_{\mathrm{cgs}}\ 10^{-3}H_{\mathrm{SI}}\langle H\rangle_{\mathrm{cgs}}$$

Remember: $[\mathrm{H}] = [1\ 0\ -2\ 0]$

cgs: $H = H_{\mathrm{cgs}}\langle H\rangle_{\mathrm{cgs}}$

$$\langle\mu_0\rangle_{\mathrm{cgs}} = 1$$

$$\rightarrow\ 4\pi\ 10^{-3}\ H_{\mathrm{SI}} = H_{\mathrm{cgs}}$$

$1\ \mathrm{A/m}$ is equivalent to $4\pi\ 10^{-3}\ \mathrm{Oe}$

The most tricky situation: dimensionless quantities

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Demagnetizing coefficients link H with M

$$\langle \mathbf{H}_\mathrm{d}(\mathbf{r})\rangle = -M_\mathrm{s}\,\bar{\bar{\mathbf{N}}}\cdot\mathbf{m}$$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V\,\mathbf{m}\cdot\bar{\bar{\mathbf{N}}}\cdot\mathbf{m}$$

Unit:

dimensionless

$$N_x + N_y + N_z = 1$$

- Definition $H = -N\,M$

$$\rightarrow \left(N_x + N_y + N_z\right)_\mathrm{cgs} = 4\pi$$

- Definition $H = -4\pi N\,M$

$$\rightarrow \left(N_x + N_y + N_z\right)_\mathrm{cgs} = 1$$

Magnetic susceptibility links M with H

- Definition $\chi = \delta M/\delta H$

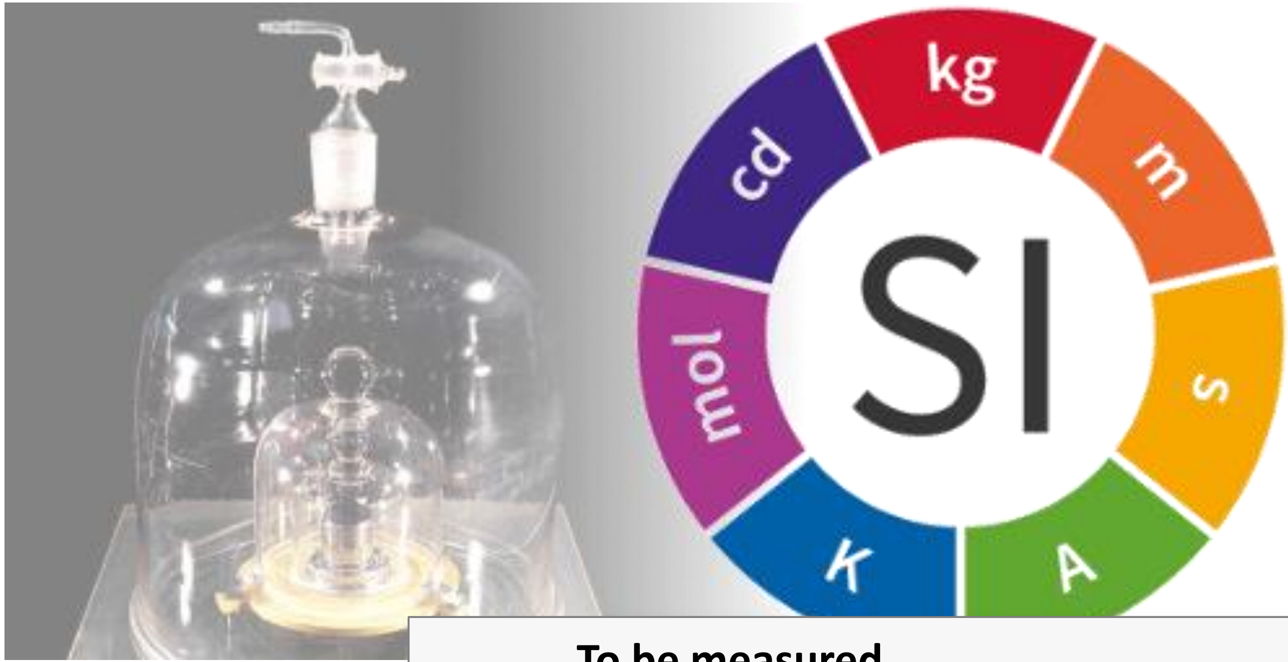
$$\rightarrow \chi_\mathrm{cgs} = \chi_\mathrm{SI}/4\pi$$

- Definition $\chi = 4\pi\,\delta M/\delta H$

$$\rightarrow \chi_\mathrm{cgs} = \chi_\mathrm{SI}$$

Both definitions are used…

Quantum revolution in SI units in 2019



Define quantities

- Times
- Length
- Mass
- Electric charge

Fixed values

- Speed of light -> Define meter
- Planck constant -> Defines kg
- Charge of the electron

To be measured

- Magnetic permeability of vacuum

$$\mu_0 \neq 4\pi \times 10^{-7}\ \text{S.I.}$$

$$\mu_0 = 4\pi[1 + 2.0(2.3) \cdot 10^{-10}] \times 10^{-7}\ \text{S.I.}$$

R. B. Goldfarb, IEEE Trans. Magn. MAG. 8, 1-3 (2017); R. B. Goldfarb, IEEE Mag. Lett. 9, 1205905 (2018)
S. Schlamminger, Redefining the kilogram and other SI units, IOP Physics World Discovery (2018)

Thank you for your attention !

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